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DEA/AHP and its application in Full Ranking of Decision Making Units

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Abstract — **The cross efficiency evaluation has long been suggested as an alternative method for ranking Decision Making Units (DMUs) in Data Envelopment Analysis (DEA). The paper aims at ranking Decision Making Units (DMUs) with a model combining Data Envelopment Analysis (DEA) and Analytical Hierarchy Process (AHP). At first, it solves a DEA model for each n of decision making units, then the DEA models results are presented in such a way that the pair-wise comparison matrix is determined and full ranking AHP model solutions are achieved. The claim would be supported with example.**

Keywords: Analytical Hierarchy Process (AHP); Data Envelopment analysis (DEA); modified cross efficiency evaluation.

1. INTRODUCTION

After introducing AHP method by Thomas L. Saaty [1, 2], finding relative weights and overall weights have always been of interest to different researchers. When pair wise comparison matrix is consistent, weights can be found easily. But when the matrix is inconsistent, the solution would be more complicated. Different methods have been proposed for finding weights. Recently, different DEA methods have helped researchers to find weights. One of them is DEAHP suggested by Ramanthan [3]. This method results in unreasonable weight factors due to not including all the available data. The main reason for this problem should be searched in not considering the relation:

$$n \geq 3(m + s)$$

Where n, m and s are number of DMUs, number of inputs and number of outputs, respectively.

Data Envelopment Analysis (DEA) is a mathematical programming technique that measures the relative efficiency of Decision Making Units (DMUs) with multiple inputs and outputs. Charnes et al [4, 5]. First proposed DEA as an evaluation tool to measure and compare the relative efficiency of DMUs. Their model assumed Constant Returns to Scale (CRS, the CCR model), the model with Variable Return to Scale (VRS, the BCC model) was developed by Banker et al [6]. The cross evaluation method was developed as a DEA extension that can be utilized to identify best performing DMUs and to rank DMUs using cross efficiency score that are liked to all DMUs (Sexton et.al [7]). The main idea of cross evaluation is to use DEA in a peer evaluation instead of a self evaluation mode. There are two principal advantages of cross evaluation: (1) It provides a unique ordering of the DMUs, and (2) It eliminates unrealistic weight restrictions from application area experts (Anderson et al [8]).

A range of models have been developed for ranking decision making units including classic and fuzzy multi-criteria decision making models and data envelopment analysis techniques. AHP method is one of the ranking techniques with some applications in weighting decision criteria. Its designing aim was subjective estimation of a set of alternatives based on various indexes or a hierarchical structure. At the top level are decision making objective and estimation indexes and at the lower level some options are in place which their estimations are based on an individual index and ultimately, diverse options are combined with respect to multiple indexes and give rise to final solution. One of the issues in relation to the AHP method application which creates some concerns among decision makers is subjective judgments in pair-wise comparison matrix. As a result, this paper tries to fill the gap using the DEA method [9]. In this way, pair-wise comparisons are not undertaken by subjective judgments rather, they are provided by powerful DEA technique and so the previous problems are eliminated. In effort to present the above mentioned model, its algorithm implementation steps are explained.

The structure of the paper is as follows: sections 2 describe the cross efficiency evaluation and Ranking with AHP method to illustrate numerical example is mentioned in section 3. The last section summarizes and concludes.

2. CROSS EFFICIENCY EVALUATION AND RANKING WITH AHP METHOD

First phase: pair-wise comparison matrix is formed using the DEA method as follows:

(a)The CCA Classic model is implemented for each n of Decision Making Units as {1,2,…,n }(model no.1):

Assuming that there are n DMUs each with m inputs and s outputs, the relative efficiency of a particular DMUk $\left(k\in\{1,2,\ldots,n\}\right)$is obtained by solving the following fractional programming problem:

$$\theta_{kk} = \max \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}}$$

$$\begin{aligned} & subject\ to: \\ & \quad \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \le 1 \qquad j = 1, 2, \dots, n \qquad\qquad (1) \\ & \quad u_{rk} \ge 0 \qquad r = 1, 2, \dots., s \\ & \quad v_{ik} \ge 0 \qquad i = 1, 2, \dots., m \end{aligned}$$

Where $\mathbf{j}$ is the DMU index $j = 1, 2, \dots, n$ the output index, $r = 1, 2, \dots, s$ and i the input index $i = 1, 2, \dots, m$, $\mathbf{y_{rj}}$ the value of the $\mathbf{r}$ th output for the $\mathbf{j}$ th DMU, x_{ij} the value of the $\mathbf{i}$ input for the $\mathbf{j}$ th DMU, u_{rk} the weight given to the $\mathbf{r}$ th output, v_{ik} the weight given to the i input. DMUk is efficient if and only if $\theta_{kk} = 1$.

DMUk selects weights that maximize its output to input ratio, subject to the constraints. A relative efficiency score of 1 indicates that the DMU under consideration is efficient, whereas a score less than 1 imply that it is inefficient. This fractional program can be converted into a linear programming problem where the optimal value of the objective function indicates the relative efficiency of DMUk. The reformulated linear programming problem, also known as the Linear CCR model, is as follows (for more detail about modified of cross efficiency method sees Yanig Ming and et.al [10]):

$$\theta_{kk}^{*} = \theta_{kk} = \max \sum_{r=1}^{s} u_{rk} y_{rk}$$

$$\begin{aligned} & subject\ to: \\ & \quad \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\ & \quad \sum_{r=1}^{s} u_{rk} y_{rk} - \sum_{i=1}^{m} v_{ik} x_{ik} \le 0 \qquad j = 1, 2, \dots, n \qquad\qquad (2) \\ & \quad u_{rk} \ge 0 \qquad r = 1, 2, \dots., s \\ & \quad v_{ik} \ge 0 \qquad i = 1, 2, \dots., m \end{aligned}$$

The modified cross efficiency evaluation:

$$
\begin{aligned}
&\theta_{kh} = \max \sum_{r=1}^{s} u_{rh} y_{rh} && h = 1,2,\ldots,n \\
&\textit{subject to}: \\
&\quad \sum_{i=1}^{m} v_{ih} x_{ih} = 1 \\
&\quad \sum_{r=1}^{s} u_{rh} y_{rj} - \sum_{i=1}^{m} v_{ih} x_{ij} \le 0 && j = 1,2,\ldots,n \qquad (3) \\
&\quad \sum_{r=1}^{s} u_{rh} y_{rk} - \theta_{kk}^{*} \sum_{i=1}^{m} v_{ih} x_{iik} = 0 \\
&\quad u_{rh} \ge 0 && r = 1,2,\ldots.,s \\
&\quad v_{ih} \ge 0 && i = 1,2,\ldots.,m
\end{aligned}
$$

With the use of above mentioned models results and following relations, a pair-wise comparison matrix as well as its a_{kh} elements is obtained:

$$a_{kh} = \frac{\theta_{kk} + \theta_{kh}}{\theta_{hh} + \theta_{hk}} \qquad k,h = 1,2,\ldots,n \qquad (4) \qquad a_{hk} = \frac{1}{a_{kh}} \qquad (5)$$

The point here is that in the AHP method, value 1 is included in diameter of pair-wise comparison matrix, and elements a_{kh} indicate the estimation of unit j with respect to k. The relation $a_{kh} > 1$ indicates lower estimate of unit j with respect to k. The pair-wise comparison matrix is completed and presented for every two units as it was said for units' j and k.

Second phase: Ranking with AHP method

The AHP method was proposed by a researcher with Iraqi origin named as Tomas Al-Saaty during 1970s. Like human brain mechanisms, the method is able to analyze various events. The AHP method enables decision makers to determine concurrent and reciprocal effects of many unknown and complex situations. This process helps decision makers to adjust some priorities based on their targets, knowledge and experiences such that they will be able to consider all of their emotions and judgments simultaneously [11]. The application of this method depends on four main steps:

Step 1. Modeling: In this step, the problem and decision making objective are delineated hierarchically based on related decision elements. Decision elements include" Decision Making Indexes" and "Decision Options".

Step 2. Preferred judgment: Multiple decision options are compared based on individual indexes and the importance of decision index is determined using pair-wise comparisons.

Step 3. Calculating relative weights: Decision elements' importance and weight are delineated in relation to each other through a set of numerical calculations.

Step 4. Combining relative weights: The step devotes to ranking decision options.

The above method algorithm is as follows:

a. Calculate the sum of each column numbers.

b. Divide each element on the column sum and obtain a new matrix which is named as normalized matrix.

c. Calculate the mean of elements in each row of normalized matrix. This mean indicates the ranking weight of each decision making unit.

3. Example

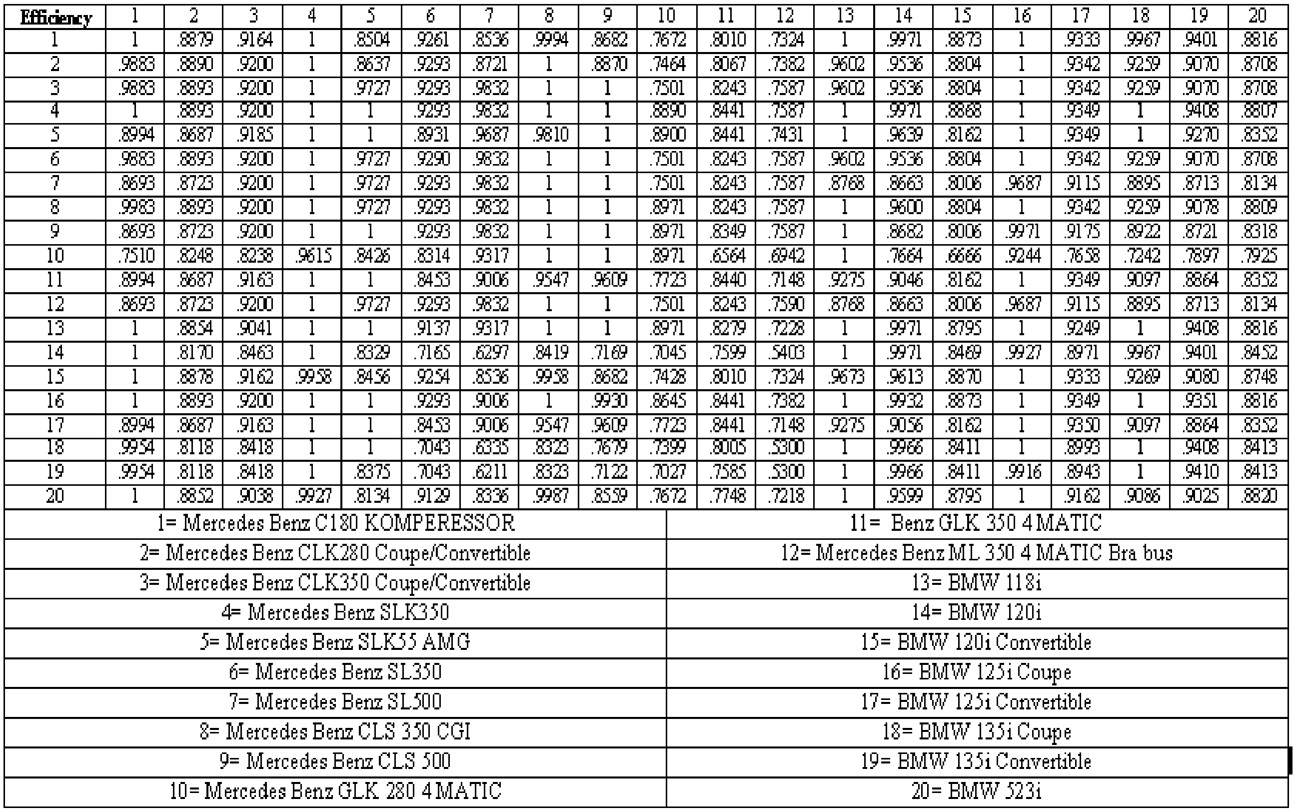
Example: Consider Table (1),…, (5), in this Tables, we have twenty DMUs with two Inputs and four outputs. Data have been taken from Iranian Cars Market... You can see the result of performance new methods for ranking DMUs in this numerical example. (Yr are outputs and Xi are inputs)

TABLE 1. TWENTY DMUS (AUTOMOBILES) WITH TWO INPUTS AND FOUR OUTPUTS

Inputs Outputs DMU	X_1= Price (Toman Currency of Iran)	X_2= Complex Oil Consumption	Y_1= Speed	Y_2=POWER-TO-WEIGHT RATIO	Y_3= Motor Capacity	Y_4= Motor Power
Mercedes Benz C180 KOMPERESSOR	75	7.5	223	103	1796	156
Mercedes Benz CLK280 Coupe/Convertible	135	9.4	231	146	2996	231
Mercedes Benz CLK350 Coupe/Convertible	145	10	250	168	3498	272
Mercedes Benz SLK350	130	9.2	250	205	3498	305
Mercedes Benz SLK55 AMG	150	12	250	228	5439	360
Mercedes Benz SL350	230	9.9	250	172	3498	315
Mercedes Benz SL500	280	11.9	250	202	5461	387
Mercedes Benz CLS 350 CGI	170	9.2	250	168	3498	292
Mercedes Benz CLS 500	220	11.7	250	211	5461	387
Mercedes Benz GLK 280 4 MATIC	120	10.4	210	125	2996	228
Mercedes Benz GLK 350 4 MATIC	130	10.7	230	149	3498	268
Mercedes Benz ML 350 4 MATIC Bra bus	230	11.5	225	117	3498	292
BMW 118i	62.2	7.4	208	101	1995	136
BMW 120i	72	7.6	213	114	1995	156
BMW 120i Convertible	95	8.2	212	103	1995	156
BMW 125i Coupe	95.8	8.7	243	147	2996	218

BMW 125i Convertible	106	9.1	236	138	2996	218
BMW 135i Coupe	113	9.6	250	196	2979	306
BMW 135i Convertible	120	9.8	250	183	2979	306
BMW 523i	104	9.3	233	121	2467	190

TABLE 2. MODIFIED CROSS EFFICIENCY EVALUATION



Efficiency	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	.8879	.9164	1	.8504	.9261	.8536	.9994	.8682	.7672	.8010	.7324	1	.9971	.8873	1	.9333	.9967	.9401	.8816
2	.9883	.8890	.9200	1	.8637	.9293	.8721	1	.8870	.7464	.8067	.7382	.9602	.9536	.8804	1	.9342	.9259	.9070	.8708
3	.9883	.8893	.9200	1	.9727	.9293	.9832	1	1	.7501	.8243	.7587	.9602	.9536	.8804	1	.9342	.9259	.9070	.8708
4	1	.8893	.9200	1	1	.9293	.9832	1	1	.8890	.8441	.7587	1	.9971	.8868	1	.9349	1	.9408	.8807
5	.8994	.8687	.9185	1	1	.8931	.9687	.9810	1	.8900	.8441	.7431	1	.9639	.8162	1	.9349	1	.9270	.8352
6	.9883	.8893	.9200	1	.9727	.9290	.9832	1	1	.7501	.8243	.7587	.9602	.9536	.8804	1	.9342	.9259	.9070	.8708
7	.8693	.8723	.9200	1	.9727	.9293	.9832	1	1	.7501	.8243	.7587	.8768	.8663	.8006	.9687	.9115	.8895	.8713	.8134
8	.9983	.8893	.9200	1	.9727	.9293	.9832	1	1	.8971	.8243	.7587	1	.9600	.8804	1	.9342	.9259	.9078	.8809
9	.8693	.8723	.9200	1	1	.9293	.9832	1	1	.8971	.8349	.7587	1	.8682	.8006	.9971	.9175	.8922	.8721	.8318
10	.7510	.8248	.8238	.9615	.8426	.8314	.9317	1	1	.8971	.6564	.6942	1	.7664	.6666	.9244	.7658	.7242	.7897	.7925
11	.8994	.8687	.9163	1	1	.8453	.9006	.9547	.9609	.7723	.8440	.7148	.9275	.9046	.8162	1	.9349	.9097	.8864	.8352
12	.8693	.8723	.9200	1	.9727	.9293	.9832	1	1	.7501	.8243	.7590	.8768	.8663	.8006	.9687	.9115	.8895	.8713	.8134
13	1	.8854	.9041	1	1	.9137	.9317	1	1	.8971	.8279	.7228	1	.9971	.8795	1	.9249	1	.9408	.8816
14	1	.8170	.8463	1	.8329	.7165	.6297	.8419	.7169	.7045	.7599	.5403	1	.9971	.8469	.9927	.8971	.9967	.9401	.8452
15	1	.8878	.9162	.9958	.8456	.9254	.8536	.9958	.8682	.7428	.8010	.7324	.9673	.9613	.8870	1	.9333	.9269	.9080	.8748
16	1	.8893	.9200	1	1	.9293	.9006	1	.9930	.8645	.8441	.7382	1	.9932	.8873	1	.9349	1	.9351	.8816
17	.8994	.8687	.9163	1	1	.8453	.9006	.9547	.9609	.7723	.8441	.7148	.9275	.9056	.8162	1	.9350	.9097	.8864	.8352
18	.9954	.8118	.8418	1	1	.7043	.6335	.8323	.7679	.7399	.8005	.5300	1	.9966	.8411	1	.8993	1	.9408	.8413
19	.9954	.8118	.8418	1	.8375	.7043	.6211	.8323	.7122	.7027	.7585	.5300	1	.9966	.8411	.9916	.8943	1	.9410	.8413
20	1	.8852	.9038	.9927	.8134	.9129	.8336	.9987	.8559	.7672	.7748	.7218	1	.9599	.8795	1	.9162	.9086	.9025	.8820

1= Mercedes Benz C180 KOMPERESSOR	11= Benz GLK 350 4 MATIC
2= Mercedes Benz CLK280 Coupe/Convertible	12= Mercedes Benz ML 350 4 MATIC Bra bus
3= Mercedes Benz CLK350 Coupe/Convertible	13= BMW 118i
4= Mercedes Benz SLK350	14= BMW 120i
5= Mercedes Benz SLK55 AMG	15= BMW 120i Convertible
6= Mercedes Benz SL350	16= BMW 125i Coupe
7= Mercedes Benz SL500	17= BMW 125i Convertible
8= Mercedes Benz CLS 350 CGI	18= BMW 135i Coupe
9= Mercedes Benz CLS 500	19= BMW 135i Convertible
10= Mercedes Benz GLK 280 4 MATIC	20= BMW 523i

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TABLE 3. PAIR-WISE COMPARISONS MATRIX OF DMUS USING DEA

DMU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1.000	1.006	1.004	1.000	.9742	1.005	1.001	1.001	.9994	1.072	1.033	1.064	1.000	1.000	1.000	1.000	1.054	1.000	1.002	.9998
2	.9940	1.000	.9998	.9998	.9379	1.000	.9491	.9998	.9486	.9498	.9901	.9975	.9278	1.016	1.064	.9998	1.011	1.002	1.025	.9958
3	.9960	1.000	1.000	1.000	.9866	1.000	1.000	1.000	1.000	.9705	.9909	.9998	.9874	1.016	.9984	1.000	1.002	1.002	1.025	1.003
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.016	1.000	.9998	1.000	1.000	1.002	1.000	.9999	1.000	.9999	1.003
5	1.026	1.066	1.014	1.000	1.000	.9955	1.007	1.004	1.000	1.086	1.000	1.007	1.000	1.073	1.048	1.000	.9999	1.000	1.083	1.082
6	.9950	1.000	1.000	1.000	1.005	1.000	.9998	.9998	.9998	.9714	1.038	.9996	.9872	1.099	.9983	.9998	1.047	1.088	1.116	1.003
7	.9990	1.054	1.000	1.000	.9930	1.000	1.000	1.000	1.000	.9478	1.036	.9998	.9629	1.137	1.025	1.027	1.032	1.146	1.187	1.047
8	.9990	1.000	1.000	1.000	.9960	1.000	1.000	1.000	1.000	1.000	1.014	.9998	1.000	1.066	.9987	1.000	1.024	1.051	1.076	1.000
9	1.001	1.054	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.017	.9998	1.000	1.090	1.026	1.002	1.011	1.070	1.132	1.054
10	.9328	1.053	1.030	.9843	.9208	1.029	1.055	1.000	1.000	1.000	.9611	1.054	1.000	.9776	.9594	.9769	.9740	.9318	1.026	1.024
11	.9681	1.010	1.009	1.000	1.000	.9634	.9653	.9862	.9833	1.040	1.000	.9845	.9691	.9952	.9835	.9999	.9999	.9740	1.018	1.014
12	.9398	1.003	1.000	1.000	.9930	1.000	1.000	1.000	1.000	.9488	1.016	1.000	.9495	1.057	.8973	.9940	1.013	1.077	1.108	.9804
13	1.000	1.078	1.014	1.000	1.000	1.013	1.039	1.000	1.000	1.000	1.032	1.053	1.000	1.000	1.014	1.000	1.034	1.000	.9999	.9998
14	1.000	.9843	.9843	1.000	.9320	.9099	.8800	.9381	.9174	1.023	1.005	.9461	1.000	1.000	.9977	.9983	1.029	.9986	.9998	1.000
15	1.000	.9398	1.002	.9980	.9542	1.002	.9756	1.001	.9747	1.042	1.017	1.114	.9862	1.002	1.000	.9998	1.039	.9852	1.007	1.000
16	1.000	1.000	1.000	1.000	1.000	1.000	.9737	1.000	.9980	1.024	1.000	1.006	1.000	1.002	1.000	1.000	.9999	1.000	1.001	.9998
17	.9488	.9891	.9980	1.000	1.000	.9551	.9690	.9766	.9891	1.027	1.000	.9872	.9671	.9718	.9625	1.000	1.000	.9713	.9924	.9844
18	1.000	.9980	.9980	1.000	1.000	.9191	.8726	.9515	.9346	1.073	1.027	.9285	1.000	1.001	1.015	1.000	1.030	1.000	.9999	1.028
19	.9980	.9756	.9756	1.000	1.000	.8961	.8425	.9294	.8834	.9747	.9823	.9025	1.000	1.000	.9930	.9990	1.008	1.000	1.000	.9988
20	1.000	1.004	.9970	.9970	.9970	.9970	.9551	1.000	.9488	.9766	.9862	1.020	1.000	1.000	1.000	1.000	1.016	.9728	1.001	1.000

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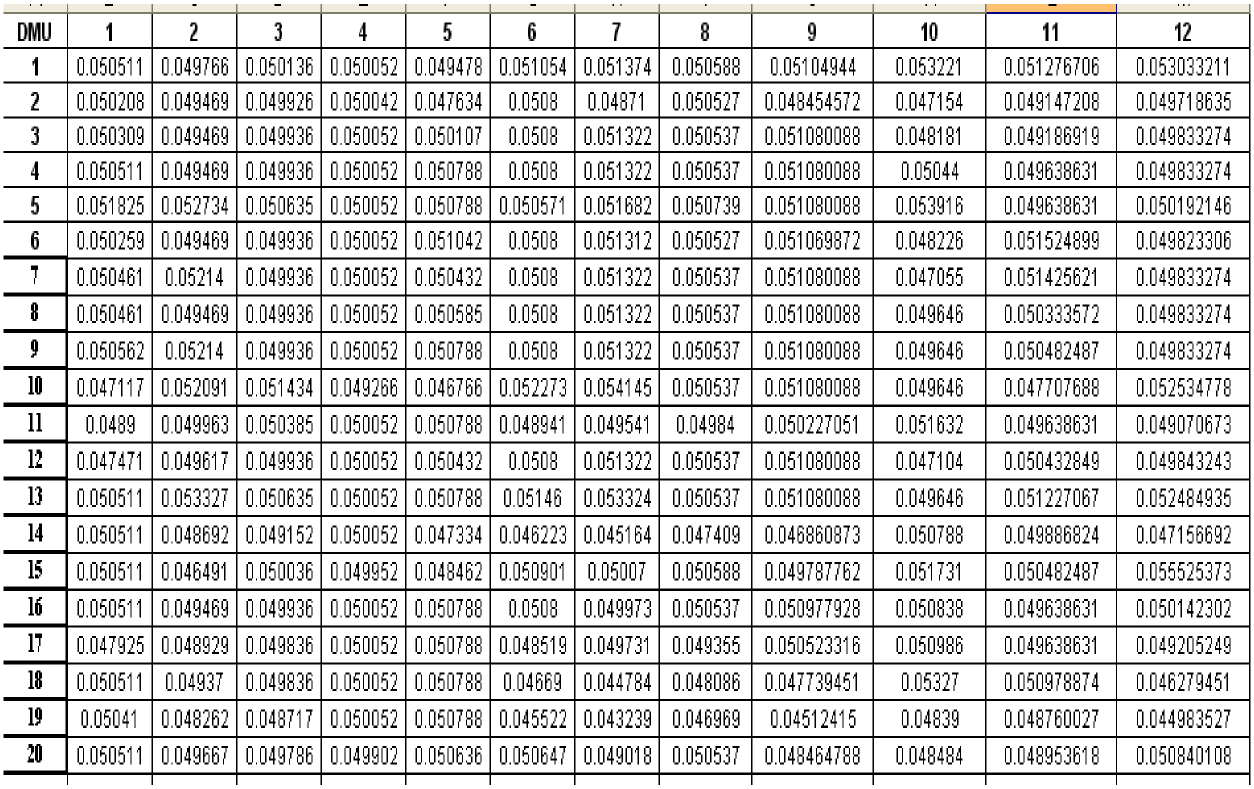
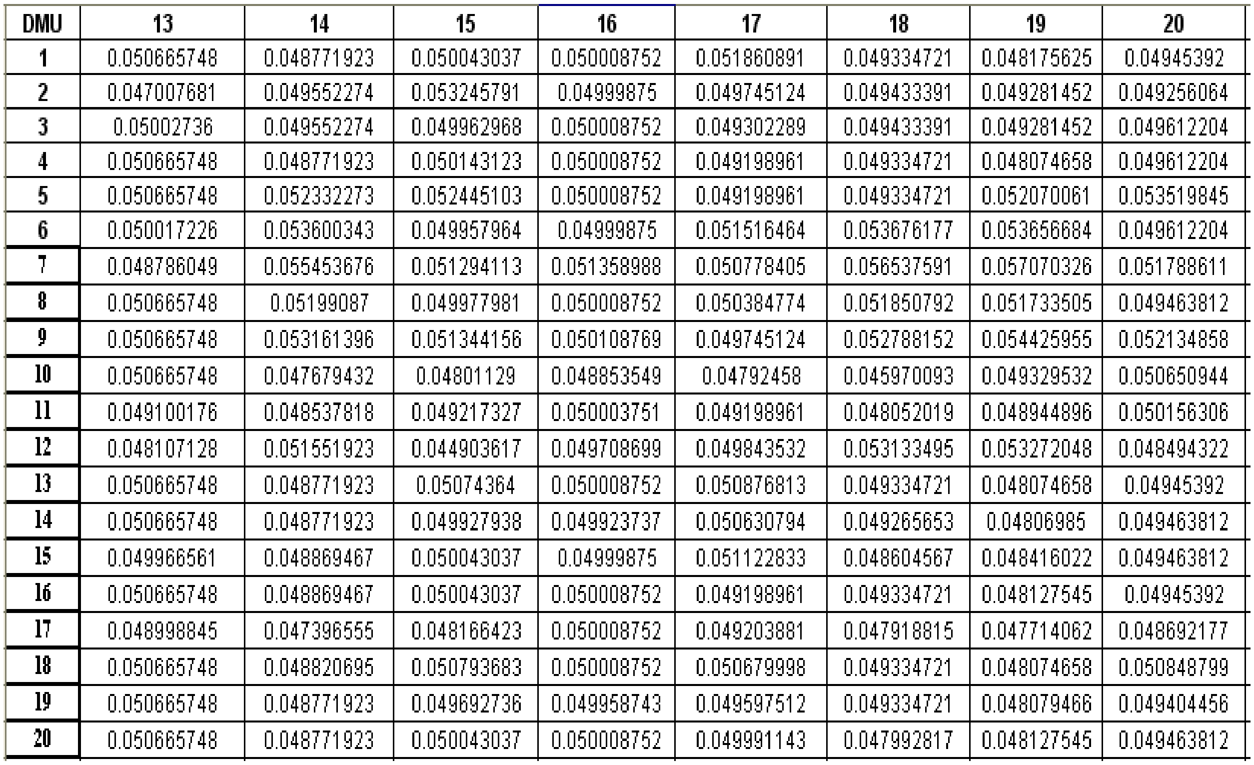
TABLE 4 . NORMALIZED MATRIX



DMU	1	2	3	4	5	6	7	8	9	10	11	12
1	0.050511	0.049766	0.050136	0.050052	0.049478	0.051054	0.051374	0.050588	0.05104944	0.053221	0.051276706	0.053033211
2	0.050208	0.049469	0.049926	0.050042	0.047634	0.0508	0.04871	0.050527	0.048454572	0.047154	0.049147208	0.049718635
3	0.050309	0.049469	0.049936	0.050052	0.050107	0.0508	0.051322	0.050537	0.051080088	0.048181	0.049186919	0.049833274
4	0.050511	0.049469	0.049936	0.050052	0.050788	0.0508	0.051322	0.050537	0.051080088	0.05044	0.049638631	0.049833274
5	0.051825	0.052734	0.050635	0.050052	0.050788	0.050571	0.051682	0.050739	0.051080088	0.053916	0.049638631	0.050192146
6	0.050259	0.049469	0.049936	0.050052	0.051042	0.0508	0.051312	0.050527	0.051069872	0.048226	0.051524899	0.049823306
7	0.050461	0.05214	0.049936	0.050052	0.050432	0.0508	0.051322	0.050537	0.051080088	0.047055	0.051425621	0.049833274
8	0.050461	0.049469	0.049936	0.050052	0.050585	0.0508	0.051322	0.050537	0.051080088	0.049646	0.050333572	0.049833274
9	0.050562	0.05214	0.049936	0.050052	0.050788	0.0508	0.051322	0.050537	0.051080088	0.049646	0.050482487	0.049833274
10	0.047117	0.052091	0.051434	0.049266	0.046766	0.052273	0.054145	0.050537	0.051080088	0.049646	0.047707688	0.052534778
11	0.0489	0.049963	0.050385	0.050052	0.050788	0.048941	0.049541	0.04984	0.050227051	0.051632	0.049638631	0.049070673
12	0.047471	0.049617	0.049936	0.050052	0.050432	0.0508	0.051322	0.050537	0.051080088	0.047104	0.050432849	0.049843243
13	0.050511	0.053327	0.050635	0.050052	0.050788	0.05146	0.053324	0.050537	0.051080088	0.049646	0.051227067	0.052484935
14	0.050511	0.048692	0.049152	0.050052	0.047334	0.046223	0.045164	0.047409	0.046860873	0.050788	0.049886824	0.047156692
15	0.050511	0.046491	0.050036	0.049952	0.048462	0.050901	0.05007	0.050588	0.049787762	0.051731	0.050482487	0.055525373
16	0.050511	0.049469	0.049936	0.050052	0.050788	0.0508	0.049973	0.050537	0.050977928	0.050838	0.049638631	0.050142302
17	0.047925	0.048929	0.049836	0.050052	0.050788	0.048519	0.049731	0.049355	0.050523316	0.050986	0.049638631	0.049205249
18	0.050511	0.04937	0.049836	0.050052	0.050788	0.04669	0.044784	0.048086	0.047739451	0.05327	0.050978874	0.046279451
19	0.05041	0.048262	0.048717	0.050052	0.050788	0.045522	0.043239	0.046969	0.04512415	0.04839	0.048760027	0.044983527
20	0.050511	0.049667	0.049786	0.049902	0.050636	0.050647	0.049018	0.050537	0.048464788	0.048484	0.048953618	0.050840108



DMU	13	14	15	16	17	18	19	20
1	0.050665748	0.048771923	0.050043037	0.050008752	0.051860891	0.049334721	0.048175625	0.04945392
2	0.047007681	0.049552274	0.053245791	0.04999875	0.049745124	0.049433391	0.049281452	0.049256064
3	0.05002736	0.049552274	0.049962968	0.050008752	0.049302289	0.049433391	0.049281452	0.049612204
4	0.050665748	0.048771923	0.050143123	0.050008752	0.049198961	0.049334721	0.048074658	0.049612204
5	0.050665748	0.052332273	0.052445103	0.050008752	0.049198961	0.049334721	0.052070061	0.053519845
6	0.050017226	0.053600343	0.049957964	0.04999875	0.051516464	0.053676177	0.053656684	0.049612204
7	0.048786049	0.055453676	0.051294113	0.051358988	0.050778405	0.056537591	0.057070326	0.051788611
8	0.050665748	0.05199087	0.049977981	0.050008752	0.050384774	0.051850792	0.051733505	0.049463812
9	0.050665748	0.053161396	0.051344156	0.050108769	0.049745124	0.052788152	0.054425955	0.052134858
10	0.050665748	0.047679432	0.04801129	0.048853549	0.04792458	0.045970093	0.049329532	0.050650944
11	0.049100176	0.048537818	0.049217327	0.050003751	0.049198961	0.048052019	0.048944896	0.050156306
12	0.048107128	0.051551923	0.044903617	0.049708699	0.049843532	0.053133495	0.053272048	0.048494322
13	0.050665748	0.048771923	0.05074364	0.050008752	0.050876813	0.049334721	0.048074658	0.04945392
14	0.050665748	0.048771923	0.049927938	0.049923737	0.050630794	0.049265653	0.04806985	0.049463812
15	0.049966561	0.048869467	0.050043037	0.04999875	0.051122833	0.048604567	0.048416022	0.049463812
16	0.050665748	0.048869467	0.050043037	0.050008752	0.049198961	0.049334721	0.048127545	0.04945392
17	0.048998845	0.047396555	0.048166423	0.050008752	0.049203881	0.047918815	0.047714062	0.048692177
18	0.050665748	0.048820695	0.050793683	0.050008752	0.050679998	0.049334721	0.048074658	0.050848799
19	0.050665748	0.048771923	0.049692736	0.049958743	0.049597512	0.049334721	0.048079466	0.049404456
20	0.050665748	0.048771923	0.050043037	0.050008752	0.049991143	0.047992817	0.048127545	0.049463812

Ultimately, DMUs rankings and weights obtained through the AHP method are presented as follows:

TABLE 5. MODELS OUTGOING (CARS RANKING)

Rank	Score	DMU
1	0.051407	Mercedes Benz SL500
2	0.051171	Mercedes Benz SLK55 AMG
3	0.051078	Mercedes Benz CLS 500
4	0.050804	Mercedes Benz SL350
5	0.05065	BMW 118i
6	0.050507	Mercedes Benz CLS 350 CGI
7	0.050493	Mercedes Benz C180 KOMPERESSOR
8	0.050051	BMW 120i Convertible
9	0.050011	Mercedes Benz SLK350
10	0.049968	BMW 125i Coupe
11	0.0499	Mercedes Benz CLK350 Coupe/Convertible
12	0.049882	Mercedes Benz ML 350 4 MATIC Bra bus
13	0.049684	Mercedes Benz GLK 280 4 MATIC

14	0.049626	BMW 523i
15	0.04961	Mercedes Benz GLK 350 4 MATIC
16	0.049466	Mercedes Benz CLK280 Coupe/Convertible
17	0.049381	BMW 135i Coupe
18	0.049179	BMW 125i Convertible
19	0.048797	BMW 120i
20	0.048336	BMW 135i Convertible

3. CONCLUSIONS

In the proposed method $n+2$ unobserved DMU are introduced. By introducing these DMus, the relation $n \geq 3(m+s)$ holds, in which n is the number of DMUs, m is the number of inputs and s is the number of outputs. The results obtained using this method shows the reality of the weights which generated. The models suggest in this paper is used to rank and evaluate of DMU s. The result seems to be logical and have economic and managerial interpretation because we use cross efficiency evaluation.

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