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Inventory Model with Multi-Item and Emergency Orders

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Abstract
In this paper, we analyze a multi-item periodic review inventory model where at each review point regular orders are placed according to an (s, S) policy and whenever the inventory level reaches a reorder level, emergency orders are placed according to (s, Q) policy. Bayesian approach is used for updating the demand. The optimum expected profit function for regular and emergency orders are derived by using the dynamic programming approach. Numerical illustration is facilitated for comparing the profit by using emergency order.

1. Introduction

Inventory management depends on a lot of information from many sources. The most important information for an inventory management system are related to the costs and the demand. Inventory theory provides the methods for minimizing the costs of the inventory system and fulfilled the demands. A periodic review method is the simple and convenient method for this purpose. There is a routine in a business organization where stock is checked at regular times, orders are placed, delivery is arranged, goods arriving are checked, and so on. The periodic review method is useful for cheap items with high demand; the stock level is also checked at specific intervals. The main advantage of periodic review method is the ease of combining orders for different items into a single order.

It is worth while to discuss the relevant past literature of multi-item inventory problems under periodic review. For a multi-period stochastic inventory problem with fixed ordering cost and linear holding and backorder costs, the (s_t, S_t) type optimal policy where s_t and S_t denote the reorder point and the order-up-to level in period t respectively, was suggested by Karlin (1960). There are quite a few articles dealing with emergency orders either in case of continuous or periodic review. Rosenshine and Obee (1976) studied the inventory system with emergency order in periodic review model. Askin (1981) used a stochastic version to determine the order interval. Snyder (1984) assumed gamma probability distribution for demand in inventory system. A mathematical analysis of the dynamic lot size model with constant cost parameters was given by Richter (1987). He discussed the stability regions for so called generalized and optimal solutions, which show how the cost input may vary, leaving the solution valid.

Algorithms, based on expected cost functions, to determine the optimal (s, S) policy for such problems are available in the literature (cf. Zheng, 1991; Zheng and Federgruen,

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1991). Chand et al. (1994) developed an inventory model with one-way substitution. The problem horizon was assumed to be infinite, and cost and demand parameters are assumed to be stationary over the problem horizon. An efficient dynamic programming algorithm was developed to find an optimal solution. Chiang and Gutierrez (1998) studied an inventory system for a periodic review policy with emergency orders. Johansen and Thorstenson (1998) analyzed an inventory model with emergency orders by assuming Poisson distribution for demand. Mohebbi and Posner (1999) considered variations of the emergency supply option for the newsboy problem.

The main concept of placing emergency order was to reduce the lead-time demand. Chiang (2001) proposed periodic review model with emergency orders and obtained the optimal policies. Kamath and Pakkala (2002) considered a gamma distribution for the demand, and applied Bayesian updating of the demand. Gutierrez et al. (2003) addressed the dynamic lot size problem with storage capacity. As in the unconstrained dynamic lot size problem, this problem admits a reduction of the state space. New properties to obtain optimal policies are introduced. Jaruphongsa et al. (2004) studied a single item, two-echelon dynamic lot-sizing model with delivery time windows, early shipment penalties, and warehouse space capacity constraints. Mitra and Chatterjee (2005) considered the lot-sizing problem in a periodic-review inventory system under non-stationary stochastic demand. They proposed a heuristic and compared its performance with the optimal solution given by dynamic programming.

Fox et al. (2006) analyzed a periodic-review inventory model where the decision maker can buy from either of two suppliers. With the first supplier, the buyer incurs a high variable cost but negligible fixed cost; with the second supplier, the buyer incurs a lower variable cost but a substantial fixed cost. Grubbstrom and Huynh (2006) developed a methodology for the case when the lead times are non-zero, whereas demand is deterministic. Kamath and Pakkala (2006) considered a periodic review inventory model with the option of placing regular orders at review time points, and emergency orders in continuous time. Ahmed et al. (2007) analyzed a finite horizon dynamic inventory model, where the objective function is a coherent risk measure. Properties of coherent risk measures allow offering a unifying treatment of risk averse and min–max type formulations. Jaruphongsa et al. (2007) presented a two-echelon dynamic lot-sizing model with two outbound delivery modes where one mode has a fixed set-up cost structure while the other has a container-based cost structure. Nahapetyan and Pardalos (2008) considered a bilinear reduction of the linear mixed integer formulation of the problem and proved that the problem is equivalent to finding a global maximum of the bilinear problem. Chan, F. T. S. and Prakesh, A. (2012) examines manufacturing SC collaboration on the basis of holding cost, backorder cost and ordering cost. Fernandes. R. et. Al. (2013) addressed the problem of inventory quantification and distribution within multi-echelon supply chains under market uncertainty and management flexibility. Helmuth, C. A. et, al. (2014) examines three key elements of study design to assess what has happened, what is currently happening, and where we should be heading as a field.

The objective of this chapter is to obtain the profit function for multi-item inventory model. The rest of the chapter is organized as follows: In next sub-section we describe a multi-item periodic review inventory model is developed. In sub-section 3 we describe N-period dynamic model. In sub-section 4, some demand distributions are taken into consideration. Some special cases are deduced in sub-section 5. In sub-section 6, we provide the expressions for the expected profit function for the dynamic models with emergency orders and without emergency orders. Numerical illustrations are given in sub-section 7. Finally conclusions are drawn in sub-section 8

2. Model Description and Notations

Consider a multi-item inventory model which is a mixture of periodic and continuous review model. Two types of orders i.e. regular orders and emergency orders are taken into consideration. In the periodic review inventory system, regular orders are placed only at discrete time points. If the time interval for random demands of items between two consecutive reviews is large, then there could be heavy shortages. Due to high shortage cost, the profit of organization becomes less. In such situation, we can place emergency orders before the end of the review period to meet the sufficient demand. The placement of emergency order is helpful to a retailer who gets items through a regular supplier for very reasonable price incurring just a minor order cost.

A periodic review policy is considered according to (s, S) policy for regular orders, at each review point, and whenever the inventory level falls below a certain level i. e. reorder point, the emergency orders are placed according to (s, Q) policy in continuous time within the review period. To model the fluctuating demand, the mean demand is considered to be a random variable, and the resulting demand distribution is updated using the Bayesian approach. Before going into the mathematical formulations, the following notations defined:

D_{ij}	Demand in i^{th} (i=1,2,…,N) review periods for j^{th} (j=1,2,…,n) Item
$F_{ij}(.)$	Density function of i^{th} (i=1,2,…,N) review periods for j^{th} (j=1,2,…,n) item
S_j	Regular order for j^{th} item
s^r_j	Reorder level for regular order for j^{th} item
Q_j	Emergency order quantity for j^{th} item
s^e_j	Reorder level for emergency order for j^{th} item
h_j	Holding cost per unit in any period for j^{th} item
C^r_{pj}	Purchase costs per unit for j^{th} item in case of regular order
C^r_{sj}	Setup costs per order for j^{th} item in case of regular order
C^e_{pj}	Purchase cost per unit for j^{th} item in case of emergency order
C^e_{sj}	Setup cost per order for j^{th} item in case of emergency order
P_j	Selling price per unit for j^{th} item
C_{2j}	Backordering cost for j^{th} item
α_j	Discount factor for j^{th} item
μ_j	Mean of the demand distribution for j^{th} item

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3. N-Period Dynamic Model

A periodic review multi-item inventory model is considered over a planning horizon consisting of N review periods and j items. The demands for j^{th} (j=1,2,…,n) items in successive review periods are independent random variables denoted by $d_{1j}, d_{2j},\ldots,d_{Nj}$ with density functions denoted by $F_{ij}(.)$, i=1,2,…,N and j=1,2,…,n. We use (s, S) periodic review inventory policy. In this policy, there is an option to continuously monitor the inventory level and place emergency orders if necessary, so that shortages are avoided or at least kept to the minimum. For emergency orders, (s, Q) policy is suitable; this may require continuous monitoring of the inventory level during the review period so that emergency order of size Q can be placed whenever the inventory level reaches s.

We consider the demand to be uniform over the review period for analytical purpose. Now we derive the objective function which is the discounted expected profit function for j^{th} item. The expected profit function is derived for a given number k (say) of emergency orders, and then the expectation over all possible k, is taken. The number of emergency orders is not fixed and is automatically determined by the policy. The dynamic programming formulation of the optimum discounted profit function in the i^{th} review period for j^{th} item has the following form:

$$P_{ij}(x_j) = \begin{cases} G_{ij}\left(S_j, Q_j, s_j^e\right) + C_{pj}^r x_j - C_{sj}^r, & if\ x_j < s_j^r \\ \\ G_{ij}\left(x_j, Q_j, s_j^e\right) + C_{pj}^r x_j, & if\ x_j \geq s_j^r \end{cases} \qquad \ldots(1)$$

Here $G_{ij}\left(S_j, Q_j, s_j^e\right)$ is the function to be maximized with respect to $S_j, Q_j\ and\ s_j^e$. Now

$$G_{ij}\left(S_j, Q_j, s_j^e\right) = \sum_{j=1}^{n}\left(-C_{pj}^r + \sum_{k=0}^{\infty} H_{kj}\left(S_j, Q_j, s_j^e\right) P[K = k]\right) \qquad \ldots(2)$$

where, $H_{kj}\left(S_j, Q_j, s_j^e\right)$ is the expected profit function of single-period for j^{th} item and the number of emergency order is k.

The expression for $H_{kj}\left(S_j, Q_j, s_j^e\right)$ for j^{th} item is obtained at the point where the inventory is exhausted. Due to the total input into the inventory in a given review period being (S_j+kQ_j), the partial emergency orders cycle for multi-items can be accounted and the costs are incurred because of the difference between this quantity and the demand over the review period for different items. Also the optimal emergency reorder level s_j^e for j^{th} item is always negative, because a positive value for s_j^e will always leave a buffer stock in the inventory which is not optimal for emergency orders particularly when shortages are allowed, and the inventory is replenished immediately.

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For k>0 and $z_j=S_j+kQ_j$ for j[th] item, the expression for $H_{kj}\left(S_j, Q_j, s_j^e\right)$ is given by

$$H_{kj}\left(S_j,Q_j,s_j^e\right)=\sum_{j=1}^{n}\left\{\int_0^{z_j} p_j u_j f_j(u_j)du_j+\int_{z_j}^{\infty}\left\{p_j z_j+\alpha p_j\left(u_j-z_j\right)\right\}f_j(u_j)du_j\right.$$
$$-k\left(C_{sj}^e+C_{pj}^e Q_j\right)-h_j\int_0^{\infty}\left(\frac{S_j^2}{2u_j}\right)f_j(u_j)du_j-C_{2j}\int_0^{\infty}\left(\frac{\left(s_j^e\right)^2}{2u_j}\right)f_j(u_j)du_j$$
$$-(k-1)\int_0^{\infty}\left\{\frac{\left(h_j\left(Q_j+s_j^e\right)^2+C_{2j}\left(s_j^e\right)^2\right)}{2u_j}\right\}f_j(u_j)du_j$$
$$-h_j\int_0^{z_j}\left\{\left(Q_j+s_j^e+z_j-u_j\right)^{t_j}\!\big/2\right\}f_j(u_j)du_j$$
$$+\alpha\int_0^{\infty}P_{(i+1)j}\left(z_j-u_j\right)f_j(u_j)du_j$$
$$\left.-\int_{z_j}^{\infty}\frac{\left\{h_j\left(Q_j+s_j^e\right)^2+C_{2j}\left(s_j^e\right)^2\right\}t_j}{2\left(Q_j+s_j^e-z_j+u_j\right)}f_j(u_j)du_j\right\}$$

...(3)

where, $t_j=1-\frac{\left(S_j-s_j^e+(k-1)Q_j\right)}{u_j}$.

Now, we evaluate the expression for $H_{kj}\left(S_j, Q_j, s_j^e\right)$ for k=0, as

$$H_0\left(S_j,Q_j,s_j^e\right)=\sum_{j=1}^{n}\left\{\int_0^{S_j}\left\{p_j u_j-h_j\left(2S_j-u_j\right)/2\right\}f_j(u_j)du_j\right.$$
$$+\alpha\int_0^{\infty}P_{(i+1)j}\left(z_j-u_j\right)f_j(u_j)du_j+\int_{S_j}^{\infty}\left\{p_j S_j+\alpha p_j\left(u_j-S_j\right)\right.$$
$$\left.\left.-\left(h_j S_j^2+C_{2j}\left(u_j-S_j\right)^2\right)/2u\right\}f_j(u_j)du_j\right\}$$

...(6B.4)

By simplifying equations (3) and (4), we evaluate the expression which also stands for k=0. Now

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$$H_{kj}\left(S_j,Q_j,s_j^e\right)=\sum_{j=1}^{n}\Bigg[a_2\int_0^{z_j}u_j f_j(u_j)du_j+(a_1+a_2)z_j\left\{1-F_j(z_j)\right\}-h_j z_j$$
$$-k\left(C_{sj}^e+C_{pj}^e Q_j\right)+a_3\mu_j-k\left[h_j\left\{\left(Q_j+s_j^e\right)^2-Q_j\left(S_j+z_j\right)\right\}\right]$$
$$-kC_{zj}\left(s_j^e\right)^2\int_0^{\infty}\left\{f_j(u_j)/2u_j\right\}du_j-a_1 z_j^2\int_z^{\infty}\left\{f_j(u_j)/u_j\right\}du_j$$
$$+\alpha\int_0^{\infty}P_{(i+1)j}\left(z_j-u_j\right)f_j(u_j)du_j\Bigg] \quad ...(5)$$

where,

$$a_1=\frac{\left(h_j+C_{2j}\right)}{2},$$
$$a_2=a_1+p_j(1-\alpha_j),$$
$$and\ a_3=\alpha_{j\,pj}-{C_{2j}}/{2}.$$

Now, by using (1), we can simplify (5) as follows:

$$H_{kj}\left(S_j,Q_j,s_j^e\right)=\sum_{j=1}^{n}a_2\int_0^{z_j}u_j f_j(u_j)du_j+(a_1+a_2)z_j\left\{1-F(z_j)\right\}-k\Big[C_{sj}^e+C_{pj}^e Q_j$$
$$+\left\{C_{2j}-h_j\left\{Q_j\left(S_j+z_j\right)-\left(Q_j+s_j^e\right)^2\right\}\right\}\int_0^{\infty}\left\{f_j(u_j)/2u_j\right\}du_j\Big]$$
$$-a_1 z_j^2\int_z^{\infty}\left\{f_j(u_j)/u_j\right\}du_j+\left(\alpha_j C_{pj}^r-h_j\right)z_j+\left(a_3-\alpha_j C_{pj}^r\right)\mu_j$$
$$+\alpha_j\left\{G_{(i+1)j}\left(S_{(i+1)j},Q_{(i+1)j},s_{(i+1)j}^e\right)-C_{sj}^r\right\}\left\{1-F_j(z_j-s_{(i+1)j}^r)\right\}$$
$$+\alpha_j\int_0^{z_j-s_{(i+1)j}^r}G_{(i+1)j}\left(z_j-u_j,Q_{(i+1)j},s_{(i+1)j}^e\right)f_j(u_j)du_j \quad ...(6)$$

Equation (6B.6) holds for i=1 to (N-1), j=1 to n and k=0, 1, 2,...

We derive the expression for the penalty costs incurred in backordered items after the planning horizon ends for the simplification of the N^{th} period for j^{th} item, as:

$$P_{N+1,j}(x)=\begin{cases}\sigma_{j1}x, & if\ x>0\\ \sigma_{j2}x, & if\ x\le 0\end{cases} \qquad ...(7)$$

The expression for profit function $\left(H_{kj}\left(S_j,Q_j,s_j^e\right)\right)$ corresponding to the Nth review period for k=0,1,2,... and j=1 to n, can be written as follows:

$$H_{kj}\left(S_j,Q_j,s_j^e\right)=\sum_{j=1}^{n}\Bigg[\left(a_2-\alpha_j(\sigma_{j1}-\sigma_{j2})\right)\int_0^{z_j}u_j f_j(u_j)du_j-(\alpha_j\sigma_{j2}+a_1+a_2)$$
$$\times z_jF(z_j)+(\alpha_j\sigma_{j2}+a_1+a_2-h_j)z_j-a_1z_j^2\int_z^{\infty}\{f_j(u_j)/u_j\}du_j$$
$$-k\left[C_{sj}^e+C_{pj}^eQ_j+\left\{C_{2j}-h_j\left\{Q_j\left(S_j+z_j\right)-\left(Q_j+s_j^e\right)^2\right\}\right\}\right]$$
$$\times\int_0^{\infty}\{f_j(u_j)/2u_j\}du_j\Bigg]$$

...(8)

Now, we calculate the probability of number of emergency orders placed for jth item.
Assume that at t_{j1}, the first emergency order for jth item is placed. Then, the rest of the demand $d_j(1-t_j)$ for jth item should be met by emergency orders. Suppose that this demand has to be met by k emergency orders, then the condition will be:

$$(k-1)Q_j<\sum_{j=1}^{n}d_j\left(1-t_j\right)\le kQ_j \qquad ...(9)$$

where $t_{j1}=\dfrac{S_j-s_j^e}{d_j}$.

Hence, the condition (6B.9) reduces to:

$$S_j-s_j^e+(k-1)Q_j<\sum_{j=1}^{n}d_j\le S_j-s_j^e+kQ_j \qquad ...(10)$$

The probability of placing k emergency orders is given by

$$P_j[K=k]=\begin{cases}F\left(S_j-s_j^e+kQ_j\right)-F\left(S_j-s_j^e+(k-1)Q_j\right), & for\,k=1,2,..\\ F\left(S_j-s_j^e\right), & for\,k=0\end{cases}$$

...(11)

The optimum expected profit function is evaluated by equation (1) with the help of equations (2), (6), (8) and (11). The expected profit function is the function of decision variables

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$S_j, Q_j \ and \ s_j^e$. We have to maximize the expected profit function, we have to find the optimal values of decision variables $S_j, Q_j \ and \ s_j^e$; for this purpose, the dynamic programming approach (backward recursion) is employed. Starting from the last period (N^{th} review period), the optimal values of decision variables $S_j, Q_j \ and \ s_j^e$ in period N are computed from equation (2), where, $H_{kj}(.)$ is given in equation (8).

4. Demand Distributions

Now we consider the periodic review model in which inventory position (items on hand plus on order) is reviewed at regular interval of time, spaced at time intervals of length *T*. At each review, if inventory position is at level, *s* or below, an order is placed for a sufficient quantity to bring inventory position upto a given level, *S*; if inventory position is above *s*, no order is placed. Since the order quantity would be larger than usual, when the demand is higher, and smaller than usual, when the demand has been less than the expectation, the order quantity is variable in size from one review to another. Because of fluctuating nature of demand we shall use the demand distributions as discussed below:

4.1 Gamma Demand Distribution

It is reasonable to assume that the demand is gamma distributed for a given value of mean demand. Probability density function of gamma distribution is given below:

$$f(x/\theta) = \frac{e^{-(x/\theta)} x^{k-1}}{\Gamma(k)\theta^k}; \qquad x>0,\ k>0,\ \theta>0. \qquad ...(12)$$

The mean and variance of gamma distribution are θk and $\theta^2 k$, respectively.

Also the square coefficient of variance is $\frac{1}{k}$.

4.2 Normal Demand Distribution

Consider the normal distribution for mean demand, which is very highly accurate approximation to other distribution. The pdf of normal distribution for demand is

$$f(x) = \frac{1}{\mu\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \qquad x>0,\ \mu \text{ and } \sigma \text{ are real constant} \qquad ...(13)$$

The mean and variance of normal distribution are μ and σ^2, respectively.

4.3 Bayesian Demand Distribution

This distribution is applied for the demand in the $(n+1)^{th}$ period. The pdf of this distribution is

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$$\left(\frac{1 + D_{n+1}}{b + \sum_{i=1}^{n} d_i} \right)^{-1} \qquad \text{...(14)}$$

Here, the parameters are (nk+a) and k.

5. Special Cases

Case 5.1: In this case we assume that no emergency order is placed at the end of planning horizon for all items, so that the expected profit function has the same structure as we obtained previously in our model for k=0. The model without emergency order in the form of dynamic programming formulation is given below:

$$P_{ij}(x) = \begin{cases} L_{ij}\left(S_j^r\right) + C_{pj}^r - C_{sj}^r & if\ x < s_j^r \\ L_{ij}\left(S_j^r\right) + C_{pj}^r & if\ x \geq s_j^r \end{cases} \qquad \text{...(15)}$$

where $S_j^r\ and\ s_j^r$ are the optimal order quantity and reorder level respectively if no emergency orders are placed.

In equation (6B.12), $L_{ij}\left(S_j^r\right)$ is given by:

$$L_{ij}\left(S_j^r\right) = -C_{pj}^r + N_{oj}(y) \qquad \text{...(16)}$$

Here $N_{0j}(y)$ is obtained as

$$N_{oj}(y) = \sum_{j=1}^{n} \left\{ \int_0^{y_j} \left\{ p_j u_j - h_j \left(2y_j - u_j\right)/2 \right\} f_j(u_j) du_j \right.$$

$$+ \alpha \int_0^{\infty} P_{(i+1)j}\left(y_j - u_j\right) f_j(u_j) du_j + \int_{S_j}^{\infty} \left\{ p_j y_j + \alpha p_j \left(u_j - y_j\right) \right.$$

$$\left. - \left(h_j y_j^2 + C_{2j}\left(u_j - y_j\right)^2\right)/2u \right\} f_j(u_j) du_j \Big\}$$

...(17)

The above equations (15) to (17) for expected profit function are same as obtained for the basic periodic review inventory model with average inventory costs.

Case 5.2: If we consider the dynamic model for single item, our model coincides with the model of Kamath and Pakkala (2006).

6. Profit Analysis

In this section, we suggest a comparison between the dynamic model with emergency orders and without emergency orders as for as expected profit function is concerned. The percentage gain in profit by using emergency orders is given by

$$\eta = \frac{(\psi_1 - \psi_2)}{\psi_1} \times 100 \qquad ...(18)$$

where ψ_1 is the profit for the model with emergency orders, and ψ_2 is the profit for the model without emergency orders.

7. Numerical Illustrations

A computational study is carried out by considering two items (i.e. j=2) in order to compare the performance of the inventory model which has the option of placing emergency order with the inventory model which does not have emergency order. The implication of better estimation of the demand distribution is also taken into account through numerical comparison between Bayesian and non-Bayesian approach. For demand Gamma demand distribution is considered. For numerical illustrations, we fix the default parameters various and costs as given below:

Parameters	α_j	P_j	C_{pj}^{r}	C_{sj}^{r}	C_{pj}^{e}	C_{sj}^{e}	h_j	C_{2j}	σ_{1j}	σ_{2j}	N_j
Item 1	0.90	3	1	5	1.5	20	0.30	5	0.1	3	2
Item 2	0.45	1.5	0.5	2.5	0.75	10	0.15	2.5	0.05	1.5	1

Table 1: Input parameters

The percentage gain profit has been computed to examine the effect of (i) purchase and setup cost of emergency orders (ii) backorder and holding cost and (iii) Bayesian updating demand.

Illustration 1. In this example, we study for the effects of purchase and setup costs of emergency orders on percentage gain in profit as displayed in table 2. It is noted from the numerical results that the percentage gain in profit decreases with the increase in purchase cost and setup cost.

$\left(C_{s1}^{e}, C_{s2}^{e}\right)$	$\left(C_{p1}^{e}, C_{p2}^{e}\right)$	% gain in profit
(5, 2.5)	(1, 0.5)	23.74
	(1.25, 0.625)	16.10
	(1.5, 0.75)	9.90
(10, 5)	(1, 0.5)	23.34
	(1.25, 0.625)	15.45
	(1.5, 0.75)	9.37
(15, 7.5)	(1, 0.5)	22.98
	(1.25, 0.625)	14.86

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	(1.5, 0.75)	8.91

Table 2: Effect of purchase cost and setup costs on percentage gain in profit.

Illustration 2. The percentage gain in profit by using the option of placing emergency orders, for various values of holding cost and backordered cost is summarized in table 3.

(h_{j1}, h_{j2})	(C_{21}, C_{22})	% gain in profit
(0.05, 0.025)	(4, 2)	5.06
	(6, 3)	16.95
	(8, 4)	38.23
(0.15, 0.75)	(4, 2)	2.61
	(6, 3)	14.57
	(8, 4)	33.31
(0.25, 0.125)	(4, 2)	2.34
	(6, 3)	12.82
	(8, 4)	30.19

Table 3: Effect of holding cost and backorder costs on percentage gain in profit.

It is observed from data that, if the shortages are backlogged immediately through emergency orders, the gain in profit increases as shortage cost increases. We also note that there is a decrease in the profit gain by increasing the holding cost; this may be due to the fact that the items ordered through the emergency channel cause additional holding costs as such there is decrease in profit gain by increasing holding cost

Illustration 3. In this illustration gamma distribution is taken into consideration, with mean μ and coefficient of variance τ. Here μ is used for as the average of the mean demand. The expected profit function is computed by using equations (1) where equations (14) are used for the demand density. The percentage gain in profit is obtained using equation (18), where, ψ_1 is the optimal values for the Bayesian approach, and ψ_2 is the optimal values for the non-Bayesian approach. The percentage gain profit by using Bayesian updating, for various values of the mean demand and coefficient of variance of the mean for two items are computed and tabulated in table 4

(τ_1, τ_2)	(μ_1, μ_2)	% gain in profit
(0.1, 0.05)	(1000, 500)	0.19
	(3000, 1500)	0.11
	(5000, 2500)	0.15
(0.2, 0.1)	(1000, 500)	0.21
	(3000, 1500)	0.14

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	(5000, 2500)	0.18
(0.3, 0.15)	(1000, 500)	0.23
	(3000, 1500)	0.17
	(5000, 2500)	0.22

Table 4: Effect of using Bayesian updating, for various values of the mean demand and coefficient of variance on percentage gain in profit.

There is an increasing trend in gain profit function with the coefficient of variance in the mean demand. It is also seen that the percentage gain in profit decreases up to a point and then increases as μ increases.

8. Conclusions

In this chapter, a multi-item inventory model which is a mixture of periodic and continuous review model has been considered for various demand distributions. The dynamic programming approach suggested provides optimal solutions for the entire time horizon. The hybrid model having the features of both periodic and continuous review situations deals with more versatile scenario of inventory systems.

Numerical examples provided for the percentage gain profit of the inventory model which has the option of placing emergency order, to inventory model which does not have emergency order, show the tractability of dynamic programming approach for rather difficulty problems.

References

1. Ahmed, S., Çakmak, U. and Shapiro, A. (2007): Coherent risk measures in inventory problems, Euro. J. Oper. Res., Vol. 182, No. 1, pp. 226-238.
2. Askin, R. G. (1981): a procedure for production lot-sizing with probabilistic dynamic demand, AIIE Trans., Vol. 13, No. 2, pp. 132-137.
3. Chand, S., Ward, J. E. and Weng, Z. K. (1994): A parts selection model with one-way substitution, Euro. J. Oper. Res., Vol. 73, No. 1, pp. 65-69.
4. Chiang, C. (2001): A note on optimal policies for a periodic inventory system with emergency orders, Comp. Oper. Res., Vol. 28, pp. 93-103.
5. Chiang, C. and Gutierrez, G. J. (1998): Optimal control policies for a periodic review inventory system with emergency orders, Naval Res. Log., Vol. 45, pp. 187-204.

6. Fox, E. J., Metters, R. and Semple, J. (2006): Optimal inventory policy with two suppliers, Oper. Res., Vol. 54, No. 2, pp. 389-393.
7. Grubbstrom, R. W. and Huynh, T. T. T. (2006): Multi-level, multi-stage capacity-constrained production–inventory systems in discrete time with non-zero lead times using MRP theory, Int. J. Prod. Eco., Vol. 101, No. 1, pp. 53-62.
8. Gutierrez, J., Sedeno-Noda, A., Colebrook, M. and Sicilia, J. (2003): A new characterization for the dynamic lot size problem with bounded inventory Comp. Oper. Res., Vol. 30, No. 3, pp. 383-395.
9. Jaruphongsa, W, Çetinkaya, S. and Lee, C. Y. (2007): Outbound shipment mode considerations for integrated inventory and delivery lot-sizing decisions, Oper. Res. Letters, Vol. 35, No. 6, pp. 813-822.
10. Jaruphongsa, W., Çetinkaya, S. and Lee, C. Y. (2004): Warehouse space capacity and delivery time window considerations in dynamic lot-sizing for a simple supply chain, Int. J. Prod. Eco., Vol. 92, No. 2, pp. 169-180.
11. Johansen, S. G. and Thorstenson, A. (1998): An inventory model with Poisson demands and emergency orders, Int. J. Prod. Eco., Vol. 56-57, pp. 275-289.
12. Kamath, K. R. and Pakkala, T. P. M. (2002): A Bayesian approach to a dynamic inventory model under an unknown demand distribution, Comp. Oper. Res., Vol. 29, No. 4, pp. 403-422.
13. Kamath, R. and Pakkala, T. P. M. (2006): A periodic review inventory model with emergency orders using bayesian demand estimation, OPSEARCH, Vol. 43, No. 4, pp. 389-403.
14. Karlin, S. (1960): Dynamic inventory policy with varying stochastic demands, Manage. Sci., Vol. 6, No. 3, pp. 231-258.
15. Mitra, S. and Chatterjee, A. K. (2005): An efficient lot-sizing heuristic for a periodic-review inventory system under non-stationary stochastic demand, OPSEARCH, Vol. 42, No. 2, pp 168-177.
16. Mohebbi, E. and Posner, M. J. M. (1999): A lost-sales continuous-review inventory system with emergency ordering, Int. J. Prod. Eco., Vol. 58, pp. 93-112.

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17. Nahapetyan, A. G. and Pardalos, P. M. (2008): A bilinear reduction based algorithm for solving capacitated multi-item dynamic pricing problems, Comp. Oper. Res., Vol. 35, No. 5, pp. 1601-1612.
18. Richter, K. (1987): Stability of the constant cost dynamic lot size model Euro. J. Oper. Res., Vol. 31, No. 1, pp. 61-65.
19. Rosenshine, M. and Obee, D. (1976): Analysis of a standing order inventory system with emergency orders, Oper. Res., Vol. 24, pp. 1143-4455.
20. Snyder, R. D. (1984): Inventory control with the gamma probability distribution, Euro. J. Oper. Res., Vol. 17, pp. 373-381.
21. Zheng, Y. (1991): A simple proof for optimalty of (s, S) policies in infinite horizon inventory systems, J. Appl. Prob., Vol. 28, pp. 802-810.
22. Zheng, Y. and Federgruen, A. (1991): Finding optimal (s, S) policies is about as simple as evaluating a single policy, Oper. Res., Vol. 39, No. 4, pp.654-665.
23. Helmuth, C. A. et, al. (2014): Supply chain management research: Key elements of study design and statistical testing, J. Ope. Manag., Available online.
24. Chan, F. T. S. and Prakesh, A. (2012): Inventory management in a lateral collaborative manufacturing supply chain: a simulation study, Inter. J. Prod. Res., Vol. 50, No. 16, pp. 4670-4685.
25. Fernandes. R. et. Al. (2013): Integrated inventory valuation in multi-echelon production/distribution systems, Inter. J. Prod. Res., Vol. 51, No. 9, pp. 2578-2592.