



## PROJECT SCHEDULING WITH RESOURCE CONSTRAINTS USING META- HEURISTIC ALGORITHMS

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### *Abstract*

In this study, the model of project timing with regard to skilled labor and solution of model using artificial bee colony (ABC) algorithm was studied. Problem examined was one project that has a certain number of activity. In this case, human labor is considered as renewable source with its number being limited. In the case of skilled labor, three skill levels are considered and each person has a real skill level. It should be noted that every person has the ability to perform their lower skills. Any activity need to complete some skills to a certain amount of man-hours. The model examined in this study is three-objective design. The first objective function is to minimize the time taken to complete the project; the second objective function is to minimize the total penalties considered for allocating employees to their lower skill levels lower and third one to minimize the total penalty for delayed project activities. In this model, it is tried that each staff member be assigned to a level closer to their skills and an amount of the penalty is considered, which in case of deviation of the actual level, it is imported into the model. To solve proposed model, meta-heuristic ABC algorithm and NSGA-II algorithm are proposed. Since the model has three conflicting objectives, ABC is designed based on Pareto archive. The results of solving problem by algorithms show that ABC has better ability to produce higher quality results than the algorithm of NSGA-II.

**Keywords:** project scheduling, multi-skill workforce; multi-objective bee colony algorithm, NSGA-II algorithm

### **I. INTRODUCTION**

Project scheduling is one of the most important issues in the field of project management. Hence, many researchers had tried to create different models to solve the problem and add new restrictions to make it closer to the real issues.

In general, organizations with are facing limited resources to carry out their projects (renewable and nonrenewable) and capital. Therefore, to be able to earn more income, it is necessary that in addition to allocating resources to activities scheduling activities to be considered. One of the resources required for the completion of projects is workers with different skills. Correct and



timely assignment of skilled labor to activities that can have a significant impact on profitability and enhance revenue.

Therefore, this study will attempt to consider Project scheduling with the assumption of skilled labor as a renewable resource. It consists of activities and every activity in the project to complete requires the renewable resources and renewable resources. The project, precedence relationships between activities is shown by the graph and it is made clear from the beginning.

In this study, human labor is considered as renewable sources, whose number is limited. In the case of skilled labor, we considered a specialty and three skill levels, the skills include: engineers, technicians and workers. Everyone has a skill. It should be noted that each person has the ability to perform its lower skills. For example, a person with senior skills has standard skills and worker's skills. Each activity to complete some skill (a skill or more) requires a certain amount of man-hours, which amount is considered as definitive.

The model examined in this study is designed with three objectives: first objective function is to minimize the time taken to complete the project; the second objective function is to minimize the total penalties for allocating employees to skill levels lower than theirs and also the third objective function just minimizes the total penalty for delay in project activities. It is necessary to mention that the first objective function model is to minimize the time required to complete the entire project and not deal with the activities individually while the third goal applies the individual activities.

In this model, the attempt is that each staff members are assigned to a level closer to their skills and in the model a value is considered as penalty, in case of deviation of the actual level, it is incorporated into model.

In this thesis, a mathematical model for the studied problem is first provided and then the bee colony algorithm based on Pareto archive is offered for solving model. The results of the solution of model by bee colony algorithm were compared with NSGA-II algorithm results with respect to the comparative indexes of multi-objective problems.

#### A) Methodology

Since the research was done on mathematical modeling and numerical solution by optimization algorithms, the present paper provides a study of theoretical and methodological research on library studies (literature and research work done previously). According to previous research study, a mathematical model was designed and an Algorithm to solve the model was chosen and a structure for it was proposed. Various problems were designed and solved by the proposed method.

#### B) Data collection

In this study, for review of literature, library studies were used. For mathematical modeling, related articles and studies were reviewed and, according to information gathered, it will be modeled. Information on optimization algorithms and comparative indices were found according to library studies. Therefore, it can be said with regard to theoretical nature of research, data collection in this study is library method.

## II. BACKGROUND

Many studies have addressed the issue of the timing of projects with limited resources (Browning and Yasin, 2009 and Tseng, 2008). Some researchers also examined the issue of the



timing of the project with regard to skilled labor as finite renewable resources (Kazemipour et al., 2012; Han Kwang Lee, 2008). But what in these studies have not been considered and is research gap is disregarding several objectives simultaneously in the models presented, among the other issues on the issue of human labor that is important as a research gap is taking into account the level of real skill and ability and inability of the labor force in some skills and taking an objective function in line with imposing penalty on the workforce to whom the skills lower than theirs are allocated, to prevent loss of skilled labor. According to the gaps, this study was conducted to fill some of the gaps, addressing the project scheduling problem by considering a multi-skilled workforce using a multi-objective algorithm.

At review of researches in this regard, Master's thesis of Zabihi can be noted. Zabihi (2015) provide project schedule taking into account the multi-skill labor force. He presented his research on the mathematical model with an objective function to minimize costs and minimize unemployment and penalties for assignment of skilled labor force to lower levels. He solved proposed model by particle swarm algorithm. The present study has differences with Zabihi (2015), which may be due to defined objective functions. In the present study, 3 objective functions to minimize completion time, minimize fines for delays in activities, and minimization of assignment of labor lower skills than theirs were considered. While in the research of Zabihi (2015), two objective functions were considered, of which only one is shared with this study. The difference of objective function of this study and research of the Zabihi (2015) caused difference in variables, parameters and constraints of the model. Meanwhile, in order to solve the bee colony algorithm based on Pareto archive was used. This is also another difference with Zabihi (2015). As can be seen in subsequent chapters, in this study, for search for solution, neighborhood operators with specific compositions were used, which case distinguishes it from research of Zabihi (2015).

### III. MATHEMATICAL MODEL

For mathematical modeling, several assumptions with limitations were considered as follows. In general assumptions about the study can be stated as follows:

- Any labor can be allocated only in their skill level or lower levels of their true skill.
- The number of workers needed for each skill to complete each activity is considered as definitive.
- The processing time for activities of the project is assumed to be definitive.
- Penalty intended to assign staff skills lower than their real skill is expressed as definitive.
- Penalty delay of activities stated definitively.
- Completion date is stated definitively.
- Activities may not be interrupted.
- Problem has several objectives.

The objective functions of model:

$$\min z_1 = c_i \quad (1)$$

Equation (1) shows the first objective function that is to minimize the time to complete the project.



$$\min z2 = \sum_{i=1}^n \sum_{s=1}^3 \sum_{k=1}^K \text{pen}(RSL_k - s)y_{ik}^s \quad (2)$$

Equation (2) represents second objective function that is to minimize the total penalties for allocating their employees to skill levels lower than theirs.

$$\min z1 = \sum_{i=1,2,\dots,n} w_i T_i \quad (3)$$

Equation (3) indicate third objective function that is to minimize the total delay penalty.

### Limitations of model:

$$c_i = \max_{\forall k,s} \{ (x_{ik}^s + p_{is} \times b_{is}) \} \forall k, s \quad \forall i \quad (4)$$

Equation (4) calculates the time to complete the activities. The completion time of an activity is maximum time to complete that activity's skills, and the time required to complete a skill, like  $s$ , is given by multiplying the required number of that skill by unit skill processing time.

$$x_{ik}^s \geq \text{ready}_i \quad (5)$$

Equation (5) is to ensure that the processing time of a larger activity is larger than presence time of the activity.

$$T_i = \max\{0, C_i - D_i\} \forall i \quad (6)$$

Equation (6) is to calculate the  $i$ -th activity delay.

$$\{ x_{ik}^s - x_{jk}^s \geq (y_{ik}^s)(y_{jk}^s)(p_{js'}) - M(f_{ijk}^{ss}) \forall i, k, s, i \neq j \}$$

$$x_{ik}^s - x_{jk}^s \geq (y_{ik}^s)(y_{jk}^s)(p_{is'}) - M(1 - f_{ijk}^{ss}) \forall i, k, s, i \neq j \quad (7)$$

Equation (7) ensures that a workforce such as labor  $k$  cannot simultaneously have two assigned activities. When  $k$ -th workforce is assigned to an activity to perform a skill, one must wait until full activity dedicated to this force is completed so that he could be allocated for doing an skill to another activity.

$$\sum_{k=1}^K (Rk_{ks} \times y_{ik}^s) = b_{is} \forall i, s \quad (8)$$

Equation (8) ensures that all activities receive labor for performing the skills they need to complete as well as any labor is assigned to his actual skill level skills or lower skills.

$$\min \{ (x_{jk}^s | y_{jk}^s = 1) - \max \{ (x_{ik}^s | y_{ik}^s = 1) \geq p_i \quad \forall k, k', s, s' \}$$

$$\forall i \geq j \text{ (} i \text{ must complete before } j \text{ in project)} \quad (9)$$

Equation (9) is to ensure compliance with prerequisite relationships between the activities of the project. If in a project,  $i$ -th activity prerequisite for  $j$ -th activity, at least the beginning of  $j$ -th activity must be more than completing time of the  $i$ -th activity (maximum start time for activity  $i$ -th plus the processing time of  $i$ -th activity).

$$x_{ik}^s \leq M y_{ik}^s \quad \forall i, k, s \quad (10)$$



Equation (10) shows in case of assignment of k-th labor to i-th activity, to do skill s, this activity can start with this assignment.

$$c_i = \max(c_i) \quad \forall i \quad (11)$$

Equation (11) calculates the time required to complete the project.

$$x_{ik}^s \geq 0 \quad \forall i, k, s \quad (12)$$

$$T_i, c_i \geq 0 \quad \forall i \quad (13)$$

$$y_{ik}^s = \{0,1\} \forall i, k, s \quad (14)$$

Relations (12), (13) and (14) were written to show the range of decision variables.

Since that particular Project scheduling problem-specific library doesn't exist considering the MSPSP and use of typical problems in PSPLIB doesn't help to compare and evaluate the efficiency of algorithms due to a change in the nature of the problems defined, a number of problems were randomly designed as examples to be solved by the algorithm.

In order to test the efficiency of the proposed algorithms, algorithms were implemented in MATLAB software environment and the results of the test problems generated by them were compared with each other according to the indicators of comparative multi-objective problems and solution time. After explaining the criteria of comparison, the experimental problems were raised and how to set parameters and finally, the results of comparisons were discussed. All calculations were done using a computer with a processor of Pentium® 4, 3.06 GHz 512 MB of RAM. The results of these tests and the exact solution obtained are displayed in the corresponding tables.

#### IV. COMPUTATIONAL RESULTS

##### Indicators of Comparison

To evaluate the effectiveness of proposed solutions, 5 indices of number of solutions on NOS, MID, quality, distribution and uniformity were used.

NOS - As mentioned, the algorithms were designed based on Pareto archive solutions and set of output solutions is the very final Pareto archive. One of the criteria is the number of solutions of the final output.

MID - This index is the sum of Euclidean distance of solutions from ideal one. In this study, the ideal solution is a matrix with two cells, the first cell value is the minimum mean value of the first objective function of solutions and the second cell value is the minimum mean value of the second objective function of solutions.

Quality Index - the index is equal to the number of Pareto solutions (non-dominated).

Uniformity index - this criteria tests uniformity of distribution of Pareto results obtained at the border of solutions. This index is defined as follows:

$$s = \frac{\sum_{i=1}^{N-1} |d_{mean} - d_i|}{(N-1) \times d_{mean}}$$

In the above equation,  $d_i$  represents the Euclidean distance between two adjacent solution was



non-dominated and  $d_{mean}$  represents mean value of  $d_i$   
Distribution index - the index is used to determine the number of non-dominated solutions found on the optimal border. Distribution index is defined as follows:

$$D = \sqrt{\sum_{i=1}^N \max(\|x_i^i - y_i^i\|)}$$

In the above equation,  $\|x_i^i - y_i^i\|$  represents the Euclidean distance between two adjacent solutions of  $x_i^i$  and  $y_i^i$  on the optimal border.

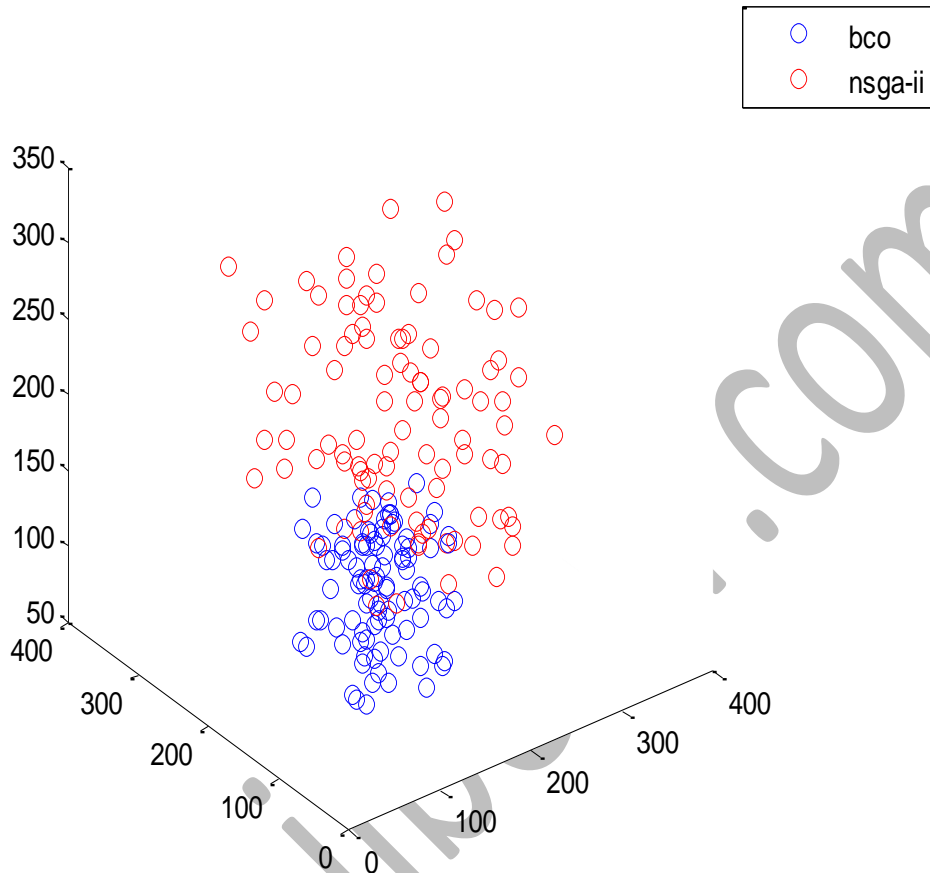
### Typical problems

Since that particular Project scheduling problem-specific library doesn't exist considering the MSPSP and use of typical problems in PSPLIB doesn't help to compare and evaluate the efficiency of algorithms due to a change in the nature of the problems defined, a number of problems were randomly designed as examples to be solved by the algorithm. Typical problems designed have been categorized in three small, medium and large sizes.

The following table shows solving the problems of small size with respect to the evaluation indexes.

Table (1): Result of solving problems with a small size

Prob.	BCO					NSGA-II				
	Quality metric	Spacing metric	Diversity metric	NOS	MID	Quality metric	Spacing metric	Diversity metric	NOS	MID
1	69.39	0.91	577.2	69	866.5	30.61	0.75	298.6	65	1196.7
2	85.3712	0.8694	633.2	65	1085.56	14.6288	0.6601	333.01	71	1265.4
3	99.011	1.003	790.6	49	1183.529	0.989	0.8649	415.5	33	1842.54
4	100	0.7634	919.5	91	863.934	0	0.99	777.1	69	1536.65
5	100	0.9911	1092.3	78	689.367	0	0.4562	879.3	84	1003.67
6	79.5204	1.3482	1213.7	53	1153.55	20.4796	0.7941	906.6	63	1807.62
7	100	0.99	1396.4	77	1067.21	0	0.73	885.1	51	1834.72
8	89.7846	0.889	1609.4	86	794.09	10.2154	0.7054	992.4	46	1659.43
9	77.04	1.05	1715.3	89	771.48	22.96	0.96	1109.6	71	1384.69
10	95.69	0.88	1779.6	103	1029.71	4.31	0.93	1414.1	69	1436.07



FIGUR (1): Pareto charts problem no. 1



This section describes the results of medium-sized and large problems (problems in the tables (2) and (3)).

Table (2): Results of solving medium-sized problems

Prob.	BCO					NSGA-II				
	Quality metric	Spacing metric	Diversity metric	NOS	MID	Quality metric	Spacing metric	Diversity metric	NOS	MID
1	77.39	1.31	1634.5	89	1009.42	22.61	0.83	1019.7	75	1655.36
2	83.55	0.49	1799.3	100	984.56	16.45	0.57	1167.4	79	1532.17
3	100	1.11	2045.1	69	1178.61	0	0.43	1372.4	78	1669.38
4	100	0.86	2629.5	105	864.55	0	0.66	1270.76	88	1957.7
5	100	0.94	2581.90	59	707.29	0	0.44	1408.6	62	2042.67
6	100	1.02	2896.6	102	1240.52	0	0.71	1753.9	74	1947.71
7	90.29	1.41	3076.4	78	853.87	9.71	0.92	1664.5	43	1237.24
8	76.93	0.69	3370.2	86	918.51	23.07	0.51	1875.3	71	1599.32
9	89.05	0.99	2847.2	97	964.71	10.95	0.77	1903.6	64	1443.65
10	100	1.3	2091.8	49	584.35	0	0.43	1564.2	27	889.45
11	91.66	0.72	2759.6	73	949.84	8.34	0.59	1409.5	67	1165.79
12	88.17	0.79	2718.9	81	779.53	11.83	0.97	1554.3	76	1329.58
13	75.83	1.05	3099.2	101	1167.75	24.17	0.56	1964.85	64	1381.33
14	100	0.51	2990.4	68	869.55	0	0.66	1549.7	50	1109.71
15	92.91	1.17	3021.6	97	1284.15	7.09	0.95	1967.9	49	1605.11

Table (3): Result of solving problems with big size

Prob.	BCO					NSGA-II				
	Quality metric	Spacing metric	Diversity metric	NOS	MID	Quality metric	Spacing metric	Diversity metric	NOS	MID
1	100	0.85	4091.2	106	1674.34	0	0.45	2085.6	90	2706.11
2	97.76	1.54	5826.1	97	1696.51	2.24	0.92	2761.2	71	2549.56
3	100	0.97	5394.5	110	1967.97	0	0.66	3549.6	84	3119.41
4	100	0.68	7499.6	122	2006.77	0	0.41	4451.1	102	3089.61
5	84.79	1.22	10230.8	99	1965.56	15.21	0.78	3722.3	99	3417.43





The results of comparison in Tables 3, 4 show ABC in all cases has higher ability to produce higher quality solutions than NSGA-II algorithm. Indicators of mean ideal solution also shows that in all cases the results produced by the bees colony algorithms are less distanced from ideal solution compared to NSGA-II algorithm. ABC is able to produce more distributed results than the NSGA-II algorithm or in other words, ABC is better able to explore and extract the possible solution region compared to the algorithm NSGA-II. As can be seen from the above tables, the NSGA-II algorithm produces solutions with more uniformity than ABC.

## V. CONCLUSION

The general results of this study can be stated as follows:

- In the context of the model:

Research in the field of project scheduling with limited resources and skilled labor have been reviewed in the second quarter. According to reviews in the second chapter, it is observed that research in the field include project scheduling with limited resources, including renewable and non-renewable in general (Kim et al., 2005; Browning and Yasin, 2010, Cheng et al. 2014). Some studies have addressed the issue of skilled labor such as Haymrl and Kvlysh (2009). In this study, the model of project scheduling taking into account the MSPSP has been conducted. In this study, the model of metaheuristic algorithm is used. Since the MSPSP project scheduling problem with limited resources is an NP-hard problem, in this study, meta-heuristic ABC based on Pareto archive is proposed to solve the model. The computational results of the NSGA-II algorithm were compared with the results of bees algorithm.

The research results and achievements based on solving algorithms are as follows:

- Bees Algorithm combined with multifunctional neighborhood search operations and structure of variable neighborhood search.
- Use of Debb Law (Debb, 2002) for the selection and acceptance of solutions at different stages of the algorithm proposed to achieve higher quality solutions, higher distribution and avoid falling into local optimum.
- In most metaheuristic algorithms, to produce initial results, randomized approaches is used, however, in this study, conventional methods available for production of solution were used.
- The results of solving examples by algorithm shows ABC in all cases has higher ability to produce higher quality solutions than NSGA-II algorithm. Indicators of distance to ideal solution also shows that in all cases the results produced by the bees colony algorithms are less distanced from ideal solution compared to NSGA-II algorithm. ABC is able to produce more distributed results than the NSGA-II algorithm or in other words, ABC is better able to explore and extract the possible solution region compared to the algorithm NSGA-II. As can be seen from the above tables, the NSGA-II algorithm produces solutions with more uniformity than ABC.
- Comparing the execution time of these problems by the two said algorithms shows that the duration of the implementation of the ABC is higher than algorithm of NSGA-II.



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