



COMPARISON OF WAGNER-WHITIN ALGORITHM RESULTS AND CUSTOM  
ECONOMIC VALUE METHOD TO DETERMINE THE TIME AND AMOUNT OF ORDER  
QUOTA WITH FUZZY NUMBERS APPROACH

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*Abstract*

*In most mid-scale industries, demand is uncertain and difficult to predict. Therefore, ordering in the right quantity and at the right time is always a decisive issue. Inventory is the storage of any item or resource used in an organization. An inventory system has a set of rules and policies that control and support the inventory level. Decides when to replenish stocks and how much to order.*

*In the present study; The author uses the economic order quantity model to obtain the optimal order quantity, then uses the Wagner-Whitin algorithm to obtain the optimal order periods, according to the order cost and maintenance cost for each case. It also uses triangular fuzzy numbers to be more accurate in calculations and to look more like real-world problems. This research includes a case study and has been done in a factory for a number of consumer goods and its raw material. The fuzzy order economic quantity model has been used to obtain the optimal order values. Then, the optimal order periods are calculated using the Wagner-Whitin algorithm. Also, to prove the superiority of Wagner-Whitin algorithm in fuzzy mode compared to definite mode, an example is presented and solved which shows this claim and the total cost obtained in fuzzy mode was much less than the total cost obtained from the algorithm in definite mode.*

*Keywords: Inventory control, Wagner\_Whitin Algorithm, Economic order quantity method, Fuzzy numbers.*



## I. INTRODUCTION

From the beginning of classical studies in the field of scientific management, which dates back to the first and second decades of the last century, the issue of inventory control and how to decide on them has been one of the concerns of managers and so far extensive efforts in this The field has been done and various models for inventory control have been presented. Most of the models that were first proposed were relatively simple models that tried to simplify the facts as much as possible and considered the model parameters with certainty, but the real world is complex and obtaining certain data and information And it is very difficult to determine. The theory of fuzzy sets, which was proposed in 1965 and has been widely developed and introduced in most scientific fields, is to solve problems in which the description of activities and events is not definitively possible and is ambiguous and uncertain. One of the issues in the field of management and decision making is subject to this ambiguity and uncertainty, is the issue of inventory control, most of the parameters of which in the real world are vague and uncertain [1].

In most industrial applications, one of the most important decisions in stock size issues is to determine the best amount of production. Today, in most industrial applications, one of the most important questions to be studied in the field of production planning and control is finding the optimal combination of using production resources to meet customer satisfaction and profitability along the planning horizon. In the real world, optimization of production planning with regard to application constraints has always been the focus of industry managers. In fact, the decision maker seeks to make a reasonable and appropriate decision in a reasonable time. The most important algorithm for determining the size of economic accumulation is Wagner-Wittin algorithm. Wagner and Whitin have developed an algorithm based on dynamic programming to determine the stock size of a single product without capacity constraints that provides the answer to the problem [2].

## II. STATEMENT OF THE PROBLEM

In most manufacturing industries, demand is uncertain and difficult to predict [3]. Calculating the optimal quantity and optimal time of ordering goods is an important and very strategic issue in any organization. Inventory refers to the storage of any commodity or resource that can be used in any organization [3]. An inventory system has a set of rules or policies that control and support the inventory level of the system [4]. It also decides when and how much reserves should be recharged [3]. The main concern of any production organization is to minimize total costs and maximize profits. In today's highly competitive market, one mistake is enough to completely destroy the organization's profits and destabilize the organization. Because capital is in circulation in the inventory control and support sector, the decisions of inventory planning and control managers are more important and strategic. Inventory costs include four costs: purchase cost, order cost, inventory maintenance cost and shortage cost [5]. Inventory planning and control managers often have to make this difficult decision considering the balance between order cost and maintenance cost [3]. Inventory planning and control managers should also pay special attention to items that have a high consumption value [5]. Inventory models also deal with determining the appropriate level of consumer goods in order to support smooth production operations [6].



The basis for deciding on an inventory model is the balance between the cost of capital to carry large amounts of inventory versus compensation for inventory shortages. The main factor influencing the choice of model is the inherent nature of demand, whether it is definite or probable. Uncertainty in demand makes inventory support and control difficult [7]. If the order quantity is low in each order period, the number of ordering times will increase and consequently the ordering cost will increase and the maintenance cost will be low and there will not be much space for storage, and sometimes there will be no storage space in The system becomes. Also, if the order quantity increases, the number of ordering times decreases and then the ordering cost decreases significantly; But maintenance costs increase. On the other hand, more storage space is needed. It should also be noted that long-term storage may cause damage to inventory items. Therefore, it is very appropriate to balance the maintenance cost with the order cost [3]. The order quantity should be such that the total cost of inventory is minimized. In fact, a suitable model is a model that considers all inventory costs at the same time and obtains the minimum total cost and provides it to decision makers. It should also be noted that classical methods and definite numbers are not very practical in the real world of industry today. In this research, Wagner-Whitin algorithm will be used and its results will be compared with the classical method of economic order value. Fuzzy numbers are also used, which have a much wider and more accurate scope for real-world problems [8].

The spatial scope of the present study is the Middle East Energy Industry Company in Arak Industrial City. This factory has started its work in the field of production of CNG compressed natural gas tanks since 2006. Its raw material is seamless chrome pipes imported from China. The main buyer of the products of this company is a group of Iran Khodro and Saipa companies and some of them are open market. The production capacity of this factory is 60,000 tanks per year.

Its time domain also includes the 6-month period of autumn and winter of 1399.

This study seeks to answer the question that how to compare the results of Wagner-Whitin algorithm and the method of economic order quantity to determine the optimal time and quantity of the order with the fuzzy number approach?

### III. IMPORTANCE AND NECESSITY OF CONDUCTING RESEARCH

In the last century, the issues of allocation and leveling of limited resources in production planning have evolved significantly. Material supply planning is a method used in the allocation of limited resources of production planning to supply parts and raw materials for final products. Following that, manufacturing resource planning and organizational resource planning have been created based on the structure of a hierarchical production plan. In these methods, the main production program, which is obtained by forecasting customer demand, is generalized to the smallest components of each product by a list of materials and parts. One of the main problems in these systems is not paying attention to resource constraints. In these systems, if the available resources are not sufficient for production, part of the production is delayed or by allocating additional required resources, the effort to execute the production plan on time is delivered. On the one hand, these delays in the production plan can cause impractical programs, and on the other hand, the allocation of additional resources by each of these systems has increased costs, which is inconsistent with the aim of reducing costs [9].



During the researches and studies in the field of industrial engineering and industrial management, it shows that the subject of "comparing the results of Wagner-Whitin algorithm and the method of economic order quantity to determine the optimal time and quantity of order with fuzzy number approach" Its introduction in the world has been neglected and neglected by the researchers and scholars of our country and it has not been paid much attention.

#### **IV. RESEARCH PURPOSES**

##### **4-1. Main purpose of research**

Comparison of Wagner-Whitin algorithm results and economical order quantity method to determine the optimal order time and quantity with fuzzy numbers approach

##### **4-2. Sub-objectives of the research**

1. To help develop the theoretical foundations of the research problem in the company under study
2. Perform Wagner-Whitin algorithm with fuzzy number approach
3. Comparison of fuzzy Wagner-Whitin algorithm results with fuzzy order economic order quantity method

##### **4-3. Research Hypothesis**

- Fuzzy Wagner-Whitin algorithm can provide an optimal method to determine the optimal order quantity and optimal order time.

#### **V. THEORETICAL LITERATURE OF RESEARCH**

##### **Wagner-Whitin algorithm**

One of the algorithms that optimizes stock size in periodic orders that has a method of solving by careful planning is the Wagner-Whitin algorithm. This algorithm has assumptions that can be adjusted by adjusting these assumptions to inventory costs according to the type of goods. The assumptions of this algorithm do not depend on the type of product and the general policy of the organization. The optimal Wagner-Whitiny method is based on planning that can get the answer with the least cost. In this method, the demand for all courses must be covered. The lengths of time periods that exist on a planning horizon are definite and equal, and orders must be issued in such a way that the products are received at the beginning of the period [2].

##### **Economic order quantity method (EOQ)**

Economic order quantity method is the simplest, most common and oldest inventory control model that determines the optimal order quantity of each product or consumable piece according to the total cost of the order and its maintenance per unit time. The main task of this model is to determine the optimal volume of orders with which the total inventory costs (maintenance and ordering) can be minimized. In this model, it is assumed that during the delivery time (this model was first introduced by Harris in 1915), all orders are delivered in full and the delivery time is specified [10].





### **Fuzzy sets**

Fuzzy logic is one of the multi-valued regions and relies on fuzzy set theory. Fuzzy sets come naturally from the generalization and expansion of definite sets. The foundation of fuzzy logic is based on the theory of fuzzy sets. This theory is a generalization of the classical theory of sets in mathematics. In classical set theory, an element is either a member of the set or not. In fact, the membership of the elements follows a zero and one and binary pattern. But fuzzy set theory extends this concept and introduces graded membership. This theory was first proposed by Professor Asgar Lotfizadeh in 1965. In this theory, the membership of the members of the set is determined by the function  $u(x)$ , where  $x$  represents a definite member and  $u$  is a fuzzy function that determines the degree of membership of  $x$  in the relevant set and its value is between zero and one [11].

## **VI. RESEARCH BACKGROUND**

### **Internal research**

Hajipour et al. (2016) in a study entitled to provide an efficient dynamic planning method to optimize the problem of order size with limited capacity, stated that maximum industrial applications is one of the most important decisions in stacking size issues to determine the best amount of production. In this paper, an integer mathematical programming model for the stack size problem is presented considering preparation time, reliability inventory, shortage cost and different production methods. The goal is to minimize the total cost of production, commissioning, maintenance, and inventory shortages. To solve the proposed model, a leading dynamic programming method is proposed and its efficiency is compared with the classical dynamic backward programming method. Finally, several experimental problems with different dimensions have been produced. Statistical analysis of the obtained computational results shows that the proposed dynamic programming method in terms of computational time is much better than classical dynamic programming [12].

Hakak Dokht et al. (2015) in a study entitled Solving the problem of determining the accumulated size of production with the possibility of transfer of startup and cost dependence on the production sequence with PSO and VDO algorithms stated, Is a product or part of production; So that the total costs of operation, production and maintenance are minimized. In this research, to develop the NP-Hard problem, determining the accumulated size with the limitation of single-stage multi-product capacity, two hypotheses of the possibility of transferring the start-up to the next period and the cost dependence on the production sequence have been added. In the continuation of MIP model, this problem is presented and solved using meta-heuristic algorithms of particle optimization and vibration damping optimization and the results are compared with the results obtained from solving the model with Lingo software [13]. Rahmani et al. (2015) during a study entitled Using dynamic planning to optimize the cost of the periodic inventory system (comparative approach with Wagner-Whitin algorithm) stated, about periodic order planning and minimizing costs related to Inventory of periodic orders, many studies have been done by researchers. This article examines this algorithm with a comparative approach and finally shows with a numerical example that this approach with the



assumptions of the type of product and the policy related to the maintenance of the product can modify the problem-solving policies by this algorithm [14].

Foreign research

Samak et al. (2013) in a research provide a model for determining the order policy that minimizes the total cost of inventory. In this paper, different inventory models such as: economical order quantity, optimal order period, minimum unit cost, minimum total cost, minimum period cost of Wagner-Whitin algorithm are reviewed. Total annual inventory costs are calculated for different sections for each model. The results obtained from each model, which are summarized for different cases, show that the algorithm and Wagner-Whitin give the optimal cost in each case [15].

Penn et al. (2009) have addressed the issue of order size with limited capacity in the closed-loop supply chain so that returned products are collected by customers. Production and reproduction capacity is limited and no drop is allowed. The general model of this problem is formulated and several useful features of this problem are identified when the cost function is concave. This problem is optimally solved using the dynamic programming algorithm [16].

Lee et al. (2007) analyzed the problem of accumulated size with limited capacity with reference and substitution products. They used the genetic algorithm to determine the start-up needs of all courses for production and reproduction, then a dynamic programming method to achieve the optimal solution by determining how much new product to produce or return products in which courses. Reworked, developed. The purpose of minimizing costs includes construction and remodeling costs, unexpected procurement costs, and start-up and maintenance costs [17].

## VII. RESEARCH METHOD

The present study is applied in terms of the purpose of the study and exploratory in terms of type, and in terms of research method is based on modeling with the help of operations research methods; The researcher will first try to identify study gaps by reviewing the literature on the subject and then will develop a model by examining the types of assumptions that are possible. In order to test the model and also to analyze and present a solution, analytical measures will be taken with the help of data collected from a case study. Finally, the proposed model and solution methods will be examined.

In order to achieve the purpose of the research, first by solving the EOQ economic order volume model, the optimal order amount is determined and then by implementing the Wagner-Whitin algorithm, the optimal amount of cost of the periodic inventory system is determined.

### **Economic order quantity model**

Fixed order quantity based on minimum maintenance costs, order costs, and shortage costs. This constant value is called the Economic Order Quantity or EOQ.



### Basic EOQ model

The economic order quantity model was first inferred by Harris in 1915. The main task of this model is to determine the optimal volume of orders that can minimize the total cost of inventories (maintenance and ordering).

This model seeks to select the amount of  $Q_i$  order for each material, so that the average cost per unit time (for example, annual) includes maintenance costs and minimum ordering costs.

The assumptions of the model are:

1. The amount of demand is known and does not change much over time.
2. Shortage of goods in stock is not allowed.
3. The time of receiving the goods (regardless of the order quantity) is fixed.
4. All ordered goods arrive at the same time.

Model parameters and variables are:

$Q_i$ : Order amount of Article  $i$  (decision variable).

$C_{hi}$ : The cost of maintaining each unit of Article  $i$  in a unit of time.

$C_{oi}$ : The cost of ordering each unit of Article  $i$

$a_i$ : The space required for each unit of Article  $i$

$D_i$ : Demand (consumption) for each unit of Article  $i$

$B_i$ : Maximum available space for storage.

$N_i$ : The number of times each product is ordered in one year

The EOQ model is formulated as follows:

$$\text{Min } C(Q) = \sum_{i=1}^I \left( \frac{C_{hi} Q_i}{2} + \frac{C_{oi} D_i}{Q_i} \right) \quad (1)$$

As:

$$0 \leq \sum_{i=1}^I a_i Q_i \leq B_i \quad (2)$$

$$n_i \leq \sum_{i=1}^I \frac{D_i}{Q_i} \leq N_i \quad (3)$$

$$Q_i > 0, \quad i = 1, 2, \dots, I \quad (4)$$

The value obtained from the order size of this method is an approximate quantity; This is because the costs of ordering and storing goods as well as the consumption rate are all approximately determined. Therefore, the decimal values of the optimal order obtained can be rounded to the nearest integer. Because the optimal ordering quantity is calculated by the cost root, the possible errors in estimating these costs will have little effect on the optimal response. Ordering and warehousing costs are inversely related, so differences in the measurement of



these costs are usually adjusted. The EOQ model is an error-free model. . In real life, some parameters of the EOQ model may be fuzzy or fuzzy random.

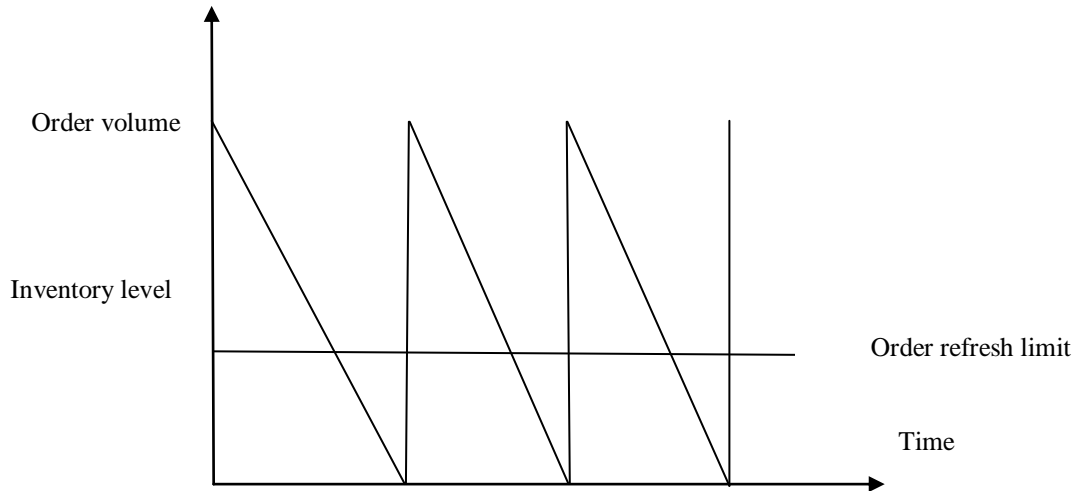


Figure 1. The main inventory system under ideal conditions

One of the important tasks of warehouse management is to obtain an optimal order point; That is, how much and at what time to request a certain type of goods or parts required for production or service, which minimizes the cost of warehousing and order costs, and on the other hand, the organization with a shortage of goods and consequently with the problem of production And do not face a lack of proper sales, or in other words, do not face the cost of shortages. The following figure shows the classic EOQ model.

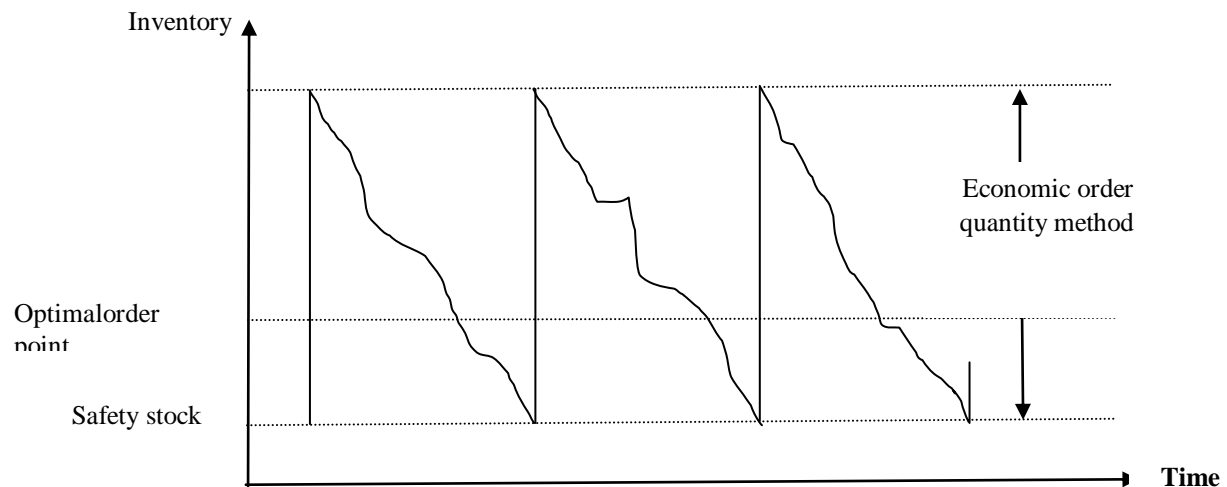
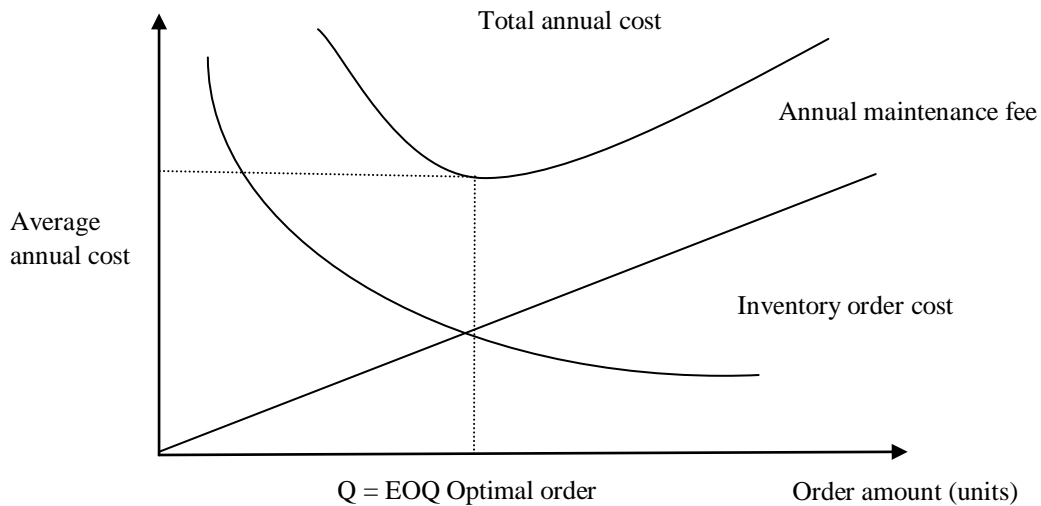
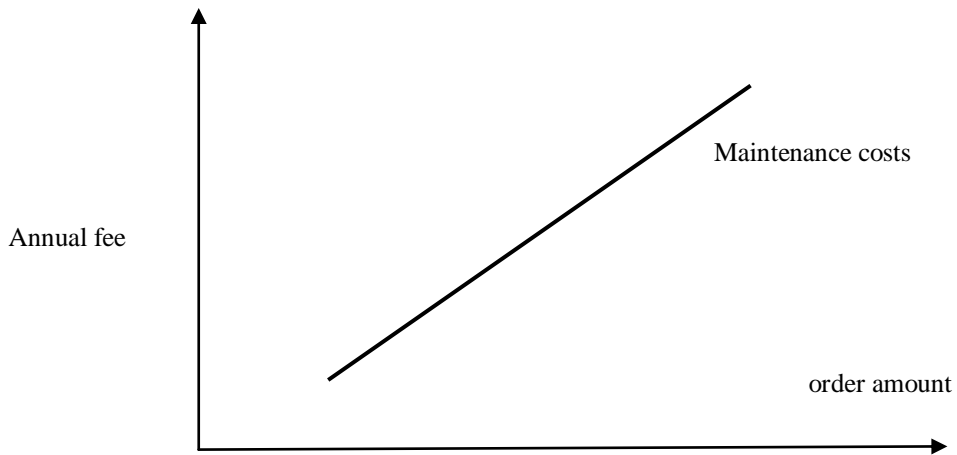
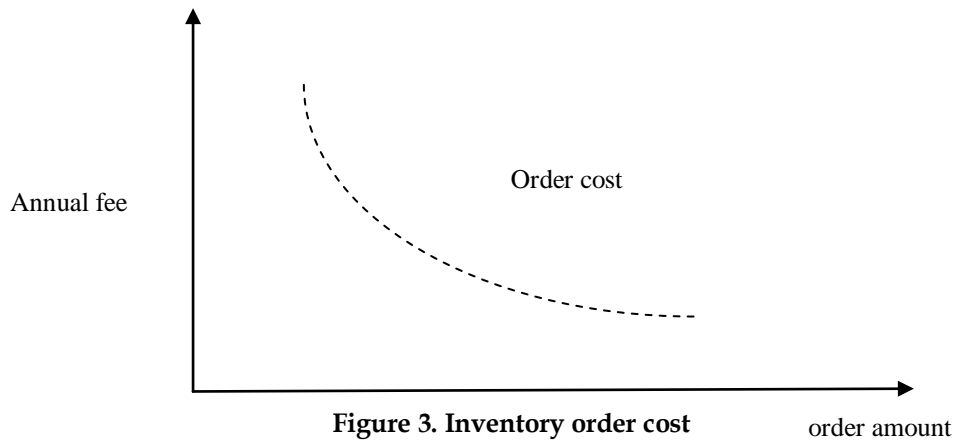


Figure 2. Inventory changes over time in the basic EOQ model







### EOQ model in fuzzy conditions

In the previous section, the EOQ model was presented in definite terms, but as mentioned, a number of model parameters may be indefinite. In fact, in the real world, parameters such as demand, maximum storage space, maximum order quantity, maintenance cost and order cost are not definite and change in different situations.

In the present study, in order to be close to the real world problems, the parameters of the model are considered as indefinite and of the triangular fuzzy number type. Therefore, the classical EOQ model is rewritten as the following fuzzy model:

$$\text{Min } C(Q) = \sum_{i=1}^I \left( \frac{\tilde{C}_{hi} Q_i}{2} + \frac{\tilde{C}_{oi} \tilde{D}_i}{Q_i} \right) \quad (5)$$

As:

$$0 \leq \sum_{i=1}^I \tilde{a}_i Q_i \leq \tilde{B}_i \quad (6)$$

$$n_i \leq \sum_{i=1}^I \frac{\tilde{D}_i}{Q_i} \leq \tilde{N}_i \quad (7)$$

$$Q_i > 0, \quad i = 1, 2, \dots, I \quad (8)$$

### Solution method

In this research, genetic algorithm has been used to solve the fuzzy order economic value model and determine the optimal order quantity. After determining the optimal order quantity, Wagner-Whitney algorithm has been implemented to optimize the cost of periodic and dynamic order inventory system.

### Genetic algorithm

Genetic algorithm is a computer science search technique to find approximate solutions to optimization and search problems. Genetic algorithms are a special type of evolutionary algorithm that uses evolutionary biology techniques such as inheritance and mutation. This algorithm was first introduced in the 20th century.

In fact, genetic algorithms use Darwin's principles of natural selection to find the optimal formula for predicting or matching the pattern. Genetic algorithms are often a good choice for regression-based prediction techniques. In short, the genetic algorithm (or GA) is a programming technique that uses genetic evolution as a problem-solving model. The problem to be solved is input, and the solutions are coded according to a pattern called the fit function. Evaluates each candidate solution, most of which are randomly selected.



### **How the chromosome is displayed**

Before a genetic algorithm can be run on a problem, a method must be used to encode the chromosomes into the computer language. One of the most common methods of coding is binary strings: strings 0 and 1. Another similar solution is to encode solutions in an array of integers or decimals, in which each position again represents an aspect of the property. In general, the way chromosomes are represented in this algorithm is done for different problems depending on the type of problem.

### **Fit function**

The fit function is calculated as a criterion for evaluating each answer. How the fit function is calculated for different problems is different.

### **Wagner-Whitin algorithm**

The performance of an operating system is directly related to and affected by the level of inventory. In this way, maintaining inventory imposes more or less costs on the organization. Excess inventory includes costs such as recession, maintenance, insurance, depreciation and other factors. The cost of maintaining excess inventory seriously reduces the financial health of an organization. But in the face of scarcity, there is the cost of losing opportunity. Therefore, the task of the organization's management is to create a balance between inventory costs in order to meet customer needs and their satisfaction with respect to reducing maintenance costs. Decision-making rules must be enforced to satisfy the projected demand for the purpose of minimum cost or maximum profit [18]. In the production discussion, inventory refers to any limited resource that is prepared and maintained to meet future needs. Inventories are the most important factors in the organization. Their importance in the organization is similar to the importance of blood in the human body. Also, due to relatively large investment, inventories need more careful planning and control [19].

Due to the importance of inventories, in the present study, we have tried to consider issues with respect to dynamic planning methods, which is a multi-step solution method and seeks the optimal mode of comparison and ultimately minimizing costs or maximizing profits. Related to minimizing inventory system costs to be used. One of the existing algorithms for solving inventory problems is Wagner-Whitin algorithm. The assumption of this algorithm is based on periodic orders. In such orders, the requirements for each period are specified, but change from period to period. Also, maintenance and ordering costs (purchase) can be different or the same for each period [18].

### **Dynamic planning**

Dynamic programming was first introduced classically by Professor Bellman in the 1960s. This method of planning includes multi-stage issues in which decisions must be made sequentially and over time, so in addition to the effects of each decision in the present will have effects for the future and therefore for future decisions. Thus, dynamic programming first involves the sequential analysis of a problem into  $n$  sub-problems, after which an optimal model is used to solve each sub-problem [20].



Like any other optimization technique, dynamic programming requires a mathematical model in which decision variables, parameters, and constraints must be clearly defined. Unlike other techniques, however, there is no single algorithm or standard framework for all dynamic programming problems. In this situation, what dynamic planning does is provide a general approach to solving this type of problem, and each problem requires its own model [21].

## VIII. FINDINGS

### Problem parameters

As mentioned earlier, the case study of the present study is the Middle East Energy Company. Therefore, the problem under investigation in this company has been solved. The planned planning has been done for 5 main consumables "wood pallet", "plastic", "glue", "color" and "seamless chrome pipe".

Table 1 shows the model parameter values. In the proposed model, a number of model parameters are considered fuzzy. A triangular fuzzy number is used to generate fuzzy values. To generate triangular numbers related to each of the fuzzy parameters ( $m_1$ ,  $m_2$ ,  $m_3$ ), first  $m_2$  is generated and then a random number  $r$  is generated in the range (0,1) and  $m_1$  is generated using the relation  $m_2 * (1-r)$  And  $m_3$  will be generated using the relation  $m_2 * (r + 1)$ . To set the fuzzy parameters  $m_2$  is randomly determined and the two values  $m_1$  and  $m_3$  are determined using the MATLAB program. For this reason, in the section for setting these parameters, we only need to mention the value of  $m_2$ .

Table 1. Model parameter values

| Seamless chrome tube | Color                     | Adhesive                 | Plastic                  | Pallet wood             | Parameter symbol | Parameter unit | Parameter description                 |
|----------------------|---------------------------|--------------------------|--------------------------|-------------------------|------------------|----------------|---------------------------------------|
| 3000                 | 850                       | 1000                     | 700                      | 1300                    | D                | Ton            | Annual demand                         |
| 50                   | 0.709                     | 0.432                    | 1.180                    | 4.625                   | Co               | \$/Ton         | Order cost                            |
| 1.23                 | 0.031                     | 0.031                    | 0.401                    | 0.154                   | Ch               | \$/Ton         | Maintenance costs                     |
| 86.31                | 2.5                       | 1.3                      | 3.25                     | 5.1                     | C                | \$             | Cost of purchasing each unit of goods |
| 6                    | 2                         | 2                        | 2                        | 6                       | A                | Time           | Number of orders per year             |
| 3                    | 0.333<br>Equal to 10 days | 0.233<br>Equal to 7 days | 0.166<br>Equal to 5 days | 0.5<br>Equal to 15 days | LT               | Month          | Delivery time                         |
| 200                  | 200                       | 200                      | 200                      | 200                     | K                | \$             | Setup cost                            |
| 35                   | 6                         | 6                        | 10                       | 10                      | ai               | Square meters  | Space required for each unit of goods |
| 175000               | 90000                     | 90000                    | 500000                   | 500000                  | B                | Cubic meters   | Maximum space available for storage   |

Also, for the implementation of the genetic algorithm, the population size is equal to 100, the number of iterations of the algorithm is equal to 200, the mutation rate is equal to 0.1 and finally, the intersection rate is equal to 0.8.



### Results tables

In this section, the results of the economical order method obtained using the genetic algorithm and the results of the Wagner-Wittin algorithm for all 5 consumables and raw materials of the company are presented.

Middle East Energy Industry Company prepares its annual orders for seamless chrome pipes in 6 periods, which will be once every three months. Also, the company, like other organizations, needs raw materials and consumable parts to produce goods and provide services and supplies them from its suppliers. The main purpose of managing the raw materials and parts required in this company is: firstly, the required goods and parts are available when needed; and secondly, the quantity of goods and parts should be appropriate, ie not as much as the cost. Impose extraordinary warehousing for the organization and occupy the space of other goods and not be so small as to stop the production line.

### Results of the economic order quantity method

As mentioned, first, using the genetic algorithm, the optimal annual order amount of the Middle East Energy Industry Company is determined. The results of this algorithm are shown in Table 2.

**Table 2. The optimal amount of economic order**

| Chrome tube | Color | Adhesive | Plastic | Pallet wood | Parameter / Item       |
|-------------|-------|----------|---------|-------------|------------------------|
| 498         | 647   | 575      | 347     | 262         | Optimal order quantity |

Also, the value of the optimal target function is calculated to be \$ 71103139 for the annual order.

### Results of Wagner-Whitin algorithm

Now, using the Wagner-Whitney algorithm, we specify periodic orders in a way that minimizes inventory maintenance costs. For each product, the results of the algorithm are shown below.

### Results of Wagner-Whitin algorithm for seamless chrome pipe

**Table 3, Table of monthly demand rate of seamless chrome pipe based on information collected in 1399**

| Ton 230 | January   |
|---------|-----------|
| 240     | February  |
| 300     | March     |
| 280     | April     |
| 270     | May       |
| 250     | June      |
| 210     | July      |
| 200     | August    |
| 190     | September |
| 290     | October   |
| 320     | November  |
| 250     | December  |





Since we order 6 times a year, the demand rate of each period is in the numbers listed in the table below:

**Table 4. Demand, maintenance and ordering of chrome pipes in each period**

| Maintenance costs | Order cost | Demand | the period        |
|-------------------|------------|--------|-------------------|
| 1.12              | 37.21      | 470    | first round       |
| 0.88              | 29.39      | 580    | The second period |
| 1.18              | 41.72      | 520    | Third period      |
| 0.98              | 43.11      | 410    | Fourth period     |
| 1.1               | 39.48      | 480    | The fifth period  |
| 1.1               | 43.29      | 570    | The sixth period  |

Ordering periods for seamless chrome pipe are once every 2 months and its delivery time is 3 months, so ordering and receiving orders will be as follows:

**Table 5. Time to register and receive seamless chrome pipe order**

| Time to receive the order | Order registration time |
|---------------------------|-------------------------|
| End of April              | First February          |
| End of June               | First April             |
| End of August             | First June              |
| End of October            | First August            |
| End of December           | First October           |
| End of February next year | First December          |



Table 6. Results of Wagner-Whitin algorithm for seamless chrome pipes in definite data mode

| Wagner-Whitin     | Demand | Order size | Order cost          | Maintenance costs  | Deficit cost |
|-------------------|--------|------------|---------------------|--------------------|--------------|
| first round       | 470    | 470        | $17489=37.21*470$   | 0                  | 0            |
| The second period | 580    | 2560       | $75238.4=2930*2560$ | $1742.4=0.88*1980$ | 0            |
| Third period      | 520    | 0          | 0                   | $1722.8=1.18*1460$ | 0            |
| Fourth period     | 410    | 0          | 0                   | $1029=0.98*1050$   | 0            |
| The fifth period  | 480    | 0          | 0                   | $627=1.1*570$      | 0            |
| The sixth period  | 570    | 0          | 0                   | 0                  | 0            |
| Total demand      | 3030   |            |                     |                    |              |
| total             |        | 3030       | 92727.4             | 5121.2             |              |

Table 6 shows the results of the Wagner-Whitin algorithm for chromium tube in 6 periods with definite parameters. As can be seen in this table, 470 units (as much as demand) are ordered in the first period, 2560 units in the second period and no orders are ordered in periods 3 to 6. The order cost is 92727.4 monetary units and the maintenance cost is equal to 5121.2 monetary units, the total cost is equal to the sum of these two and equal to 97848.6 monetary units.

As mentioned, in this study, the parameters are considered as triangular fuzzy numbers (TFN). The following tables show the fuzzy values of the parameters.

Table 7. Fuzzy and diffuse values demand for seamless chrome pipes

| Rounded values | Default values of demand after ranking | Demand             | the period        |
|----------------|--|--------------------|-------------------|
| 355            | 355.124                                | (287.1,470,852.9)  | first round       |
| 422            | 422.39                                 | (254.6,580,1105.4) | The second period |
| 500            | 500.19                                 | (245.4, 520,586)   | Third period      |
| 298            | 297.65                                 | (235.2,410,784.5)  | Fourth period     |
| 389            | 388.94                                 | (176.5,480,783.5)  | The fifth period  |
| 553            | 553.32                                 | (514.4,570,625.6)  | The sixth period  |



Table 8. Fuzzy and diffuse values of costs for seamless chrome pipes

| Default values of order cost after ranking | Order cost          | Diffuse values of maintenance cost after ranking | Maintenance costs   | the period        |
|--|---------------------|--|---------------------|-------------------|
| 26.53                                      | (1.59,37.21,72.82)  | 1.026  | (0.808,1.12,1.43)   | first round       |
| 25.11                                      | (15.12,29.39,43.65) | 0.736  | (0.399,0.88,1.36)   | The second period |
| 31.70                                      | (8.33,41.72,75.11)  | 0.841  | (0.0501,1.18,2.309) | Third period      |
| 41.28                                      | (36.99,43.11,49.22) | 0.696  | (0.034,0.98,1.93)   | Fourth period     |
| 34.48                                      | (22.83,39.48,56.13) | 1.048  | (0.93,1.1,1.27)     | The fifth period  |
| 31.39                                      | (3.65,43.29,82.93)  | 0.779  | (0.032,1.1,2.17)    | The sixth period  |

After solving the Wagner-Whitin algorithm with fuzzy parameters, the results of the algorithm are shown in Table 9.

Table 9. Results of Wagner-Whitin algorithm with fuzzy parameters for seamless chrome pipes

| Wagner-Whitin     | Demand | Order size | Order cost          | Maintenance costs    | Deficit cost |
|-------------------|--------|------------|---------------------|----------------------|--------------|
| first round       | 355    | 355        | $9418.1=26.53*355$  | 0                    | 0            |
| The second period | 422    | 2162       | $287.82=25.11*2162$ | $1280.64=0.736*1740$ | 0            |
| Third period      | 500    | 0          | 0                   | $1042.84=0.841*1240$ | 0            |
| Fourth period     | 298    | 0          | 0                   | $655.632=0.696*942$  | 0            |
| The fifth period  | 389    | 0          | 0                   | $579.544=1.048*553$  | 0            |
| The sixth period  | 553    | 0          | 0                   | 0                    | 0            |
| Total demand      | 2517   |            |                     |                      |              |
| total             |        | 2517       | 63705.92            | 3557.816             |              |



Table 9 shows the results of the Wagner-Whitin algorithm for a chromium tube with fuzzy parameters in 6 periods. As can be seen in this table, in the first period, 355 units (as much as demand) are ordered, and in the second period, 2162 units of goods are ordered, and in periods 3 to 6, orders are not ordered. The order cost is 63705.92 monetary units and the maintenance cost is equal to 3557.816 monetary units, the total cost is equal to the sum of these two and equal to 67263.736 monetary units.

Comparing the results of Tables 6 and 9, we see that the order cost, maintenance cost and total cost in the fuzzy mode of the Wagner-Whitin algorithm are significantly less than the definite mode.

#### Results of Wagner-Whitin algorithm for wood pallets

In the following, Wagner-Whitin algorithm for wood pallets with only fuzzy parameters that is the purpose of this dissertation is presented.

**Table 10. Fuzzy and defuzzy parameters Demand and costs for pallets**

| Defazi amount of maintenance cost | Maintenance costs   | Defazi amount of order cost | Order cost        | Rounded amounts of demand | Default values of demand after ranking | Demand                 | the period        |
|-----------------------------------|---------------------|-----------------------------|-------------------|---------------------------|--|------------------------|-------------------|
| 0.15                              | (0.14,0.154,0.17)   | 3.28                        | (1.35,4.11,6.87)  | 147                       | 147.2                                  | (43.55,191,338.448)    | first round       |
| 0.093                             | (0.052,0.11,0.17)   | 2.48                        | (0.825,3.19,5.55) | 158                       | 157.86                                 | (14.77,220,425.228)    | The second period |
| 0.049                             | (0.027,0.058,0.089) | 3.55                        | (2.021,4.21,6.40) | 223                       | 223.1                                  | (8.586,315,621.413)    | Third period      |
| 0.089                             | (0.017,0.12,0.22)   | 2.48                        | (1.81,2.77,3.73)  | 198                       | 198.23                                 | (169.674,210,250.326)  | Fourth period     |
| 0.13                              | (0.077,0.15,0.22)   | 3.93                        | (3.50,4.12,4.74)  | 173                       | 172.8                                  | (150.0026,180,204.997) | The fifth period  |
| 0.088                             | (0.061,0.1,0.14)    | 3.29                        | (1.65,3.99,6.33)  | 157                       | 157.31                                 | (60.443,199,337.557)   | The sixth period  |



Table 11. Results of Wagner-Whitin algorithm with fuzzy parameters for wood pallets

| Wagner_Whitin     | Demand | Order size | Order cost         | Maintenance costs  | Deficit cost |
|-------------------|--------|------------|--------------------|--------------------|--------------|
| first round       | 147    | 147        | $482.16=3.28*147$  | 0                  | 0            |
| The second period | 158    | 381        | $944.88=2.48*381$  | $20.739=0.093*223$ | 0            |
| Third period      | 223    | 0          | 0                  | 0                  | 0            |
| Fourth period     | 198    | 528        | $1309.44=2.48*528$ | $29.37=0.089*330$  | 0            |
| The fifth period  | 173    | 0          | 0                  | $20.41=0.13*157$   | 0            |
| The sixth period  | 157    | 0          | 0                  | 0                  | 0            |
| Total demand      | 1056   | 0          | 0                  | 0                  | 0            |
| Total             |        | 1056       | 2736.93            | 70.519             |              |

Table 11 shows the results of Wagner-Whitin algorithm for wood pallets with fuzzy parameters in 6 periods. As can be seen in this table, in the first period, 147 units (as much as demand) are ordered and in the second period, 381 units of goods are ordered. In the third period, the order is not placed and in the fourth period, 528 units of goods are ordered. Also not ordered in courses 5 and 6. The order cost is 2736.93 monetary units and the maintenance cost is equal to 70.519 monetary units, the total cost is equal to the sum of these two and equal to 2807.449 monetary units.





Results of Wagner-Whitin algorithm for plastics

Note that plastic is only ordered twice a year.

**Table 12. Demand and cost parameters for plastics**

| Defazi amount of maintenance cost | Maintenance costs | Defazi amount of order cost | Order cost        | Rounded amounts of demand | Default values of demand after ranking | Demand        | the period        |
|-----------------------------------|-------------------|-----------------------------|-------------------|---------------------------|--|---------------|-------------------|
| 0.388                             | (0.36,0.4,0.44)   | 1.45                        | (1.062,1.18,1.29) | 398                       | 397.7                                  | (369,410,451) | first round       |
| 0.514                             | (0.477,0.53,0.58) | 1.30                        | (1.206,1.34,1.47) | 301                       | 300.7                                  | (279,310,341) | The second period |

**Table 13. Results of Wagner-Whitin algorithm with fuzzy parameters for plastic pallet**

| Wagner-Whitin     | Demand | Order size | Order cost       | Maintenance costs | Deficit cost |
|-------------------|--------|------------|------------------|-------------------|--------------|
| first round       | 398    | 398        | $577.1=1.45*398$ | 0                 | 0            |
| The second period | 301    | 301        | $391.3=1.30*301$ | 0                 | 0            |
| Total demand      | 699    | 0          | 0                | 0                 | 0            |
| Total             |        | 699        | 968.4            | 0                 |              |

Table 13 shows the results of the Wagner-Whitin algorithm for plastic pallets with fuzzy parameters in 2 periods. As can be seen in this table, in the first period, 398 units (as much as demand) are ordered and in the second period, 301 units of goods (as much as demand) are ordered. The order cost is 968.4 monetary units and the maintenance cost is equal to 0 monetary units. In fact, according to the results of this algorithm, plastic pallets are ordered in each period and maintenance costs are not included in this product.



Results of Wagner-Whitin algorithm for adhesive

**Table 14. Demand and cost parameters for the adhesive**

| Defazi amount of maintenance cost | Maintenance costs  | Defazi amount of order cost | Order cost         | Rounded amounts of demand | Default values of demand after ranking | Demand        | the period        |
|-----------------------------------|--------------------|-----------------------------|--------------------|---------------------------|--|---------------|-------------------|
| 0.0291                            | (0.027,0.03,0.033) | 0.417                       | (0.387,0.43,0.473) | 534                       | 533.5                                  | (495,550,605) | first round       |
| 0.0194                            | (0.018,0.02,0.022) | 0.388                       | (0.36,0.4,0.44)    | 466                       | 465.6                                  | (432,480,528) | The second period |

**Table 15. Results of Wagner-Whitin algorithm with fuzzy parameters for adhesive**

| Wagner-Whitin     | Demand | Order size | Order cost          | Maintenance costs | Deficit cost |
|-------------------|--------|------------|---------------------|-------------------|--------------|
| first round       | 534    | 1000       | $13.561=0.0291*466$ | 0                 | 0            |
| The second period | 466    | 0          | 0                   | 0                 | 0            |
| Total demand      | 1000   | 0          | 0                   | 0                 | 0            |
| Total             |        | 417        | 13.561              | 0                 |              |

Table 15 shows the results of the Wagner-Whitin algorithm for adhesives with fuzzy parameters in 2 periods. As can be seen in this table, in the first period, 1000 units are ordered and in the second period, goods are not ordered. The order cost is 417 monetary units and the maintenance cost is equal to 13.561 monetary units. The total cost of this product is 430,561 monetary units.



Results of Wagner-Whitin algorithm for color

Table 16. Demand and cost parameters for color

| Defazi amount of maintenance cost | Maintenance costs  | Defazi amount of order cost | Order cost         | Rounded amounts of demand | Default values of demand after ranking | Demand            | the period        |
|-----------------------------------|--------------------|-----------------------------|--------------------|---------------------------|--|-------------------|-------------------|
| 0.301                             | (0.341·0.31·0.279) | 0.688                       | (0.78·0.709·0.638) | 437                       | 436.5                                  | (495·405·405)     | first round       |
| 0.301                             | (0.341·0.31·0.279) | 0.679                       | (0.77·0.7·0.63)    | 403                       | 402.55                                 | (456.5·415·373.5) | The second period |

Table 17. Results of Wagner-Whitin algorithm with fuzzy parameters for color

| Wagner-Whitin     | Demand | Order size | Order cost         | Maintenance costs   | Deficit cost |
|-------------------|--------|------------|--------------------|---------------------|--------------|
| first round       | 437    | 840        | $577.92=0.688*840$ | $121.303=0.301*403$ | 0            |
| The second period | 403    | 0          | 0                  | 0                   | 0            |
| Total demand      | 840    | 840        | 0                  | 0                   | 0            |
| Total             |        |            | 577.92             | 121.303             |              |

Table 17 shows the results of the Wagner-Whitin algorithm for color with fuzzy parameters in 2 periods. As can be seen in this table, in the first period, 840 units are ordered and in the second period, goods are not ordered. The order cost is 577.92 monetary units and the maintenance cost is equal to 121.303 monetary units. The total cost for this product is equal to 699,223 monetary units.

## IX. CONCLUSION

Much research has been done by researchers on periodic order planning and minimizing costs associated with periodic order inventories. One of the algorithms for optimizing the stock size in periodic orders, which has a solution method by dynamic programming, is the Wagner-Wittin algorithm. This algorithm has assumptions that can be adjusted by adjusting these assumptions to inventory costs according to the type of goods. The assumptions of this algorithm do not depend on the type of product and the general policy of the organization.



The performance of an operating system is directly related to and affected by the level of inventory. In this way, maintaining inventory imposes more or less costs on the organization. Excess inventory includes costs such as recession, maintenance, insurance, depreciation and other factors. The cost of maintaining excess inventory seriously reduces the financial health of an organization. But in the face of scarcity, there is the cost of losing opportunity. Therefore, the task of the organization's management is to create a balance between inventory costs to be able to meet customer needs and their satisfaction with respect to reducing maintenance costs. Decision-making rules must be enforced to satisfy the projected demand for the purpose of minimum cost or maximum profit [18].

Due to the importance of inventories, the present study was formed with the aim of "determining the optimal order amount and optimizing the cost of the periodic inventory system with the Wagner-Whitin algorithm". In this research, first, the economic order determination (EOQ) model with fuzzy parameters to determine the optimal amount of economic order has been solved. In this regard, the meta-heuristic algorithm of genetic algorithm has been used. To implement this algorithm, the main structure of the algorithm has been combined with an improvement procedure based on the variable neighborhood search structure (VNS).

Considering that the case study of the present study is the Middle East Energy Industry Company, the problem studied in this company has been solved and the planned planning for 5 consumables "wood pallet", "plastic", "glue", "color" And "seamless chrome pipe".

After determining the optimal amount of economic order for the 5 mentioned goods, Wagner-Whitin algorithm has been used to determine periodic orders based on optimization of inventory and order costs.

To run the Wagner-Whitin algorithm, the demand for courses, maintenance and ordering costs are considered fuzzy. It should also be noted that this algorithm for seamless chrome pipe has been implemented in both definite and fuzzy mode for 6 periods per year. The Wagner-Whitin algorithm with fuzzy parameters was also implemented for wood for 6 periods per year, and for the other three products, which are ordered only twice a year, was examined in two items.

The results of the present study are as follows:

- Design and implementation of genetic algorithms to solve fuzzy EOQ models.
- Based on solving the fuzzy EOQ model by genetic algorithm, the optimal economic order value for "wood pallet", "plastic", "glue", "color" and "seamless chrome pipe" goods is equal to 262, 347, 575, 647, respectively. And 498 tons per designated order period.

Based on the results of solving the fuzzy EOQ model by the genetic algorithm, the optimal value of the model objective function is calculated to be \$ 6711.3139 for the annual order.

- Based on the results of Wagner-Whitin algorithm with definite parameters for seamless chrome pipe product, the first period, 470 units (as much as demand) is ordered, and in the second period, 2560 units of goods and in periods 3 to 6, custom Cannot be given. The order



cost is 92727.4 monetary units and the maintenance cost is equal to 5121.2 monetary units, the total cost is equal to the sum of these two and equal to 97848.6 monetary units.

- Based on the results of Wagner-Whitin algorithm with fuzzy parameters for seamless chrome pipe product, the first period, 355 units (as much as demand) is ordered and in the second period, 2162 units of goods and in periods 3 to 6, custom Cannot be given. The order cost is 63705.92 monetary units and the maintenance cost is equal to 3557.816 monetary units, the total cost is equal to the sum of these two and equal to 67263.736 monetary units.

- Based on the results of the Wagner-Whitin algorithm with fuzzy parameters for wood pallets, in the first period, 147 units (as much as demand) are ordered and in the second period, 381 units of goods are ordered. In the third period, the order is not placed and in the fourth period, 528 units of goods are ordered. Also not ordered in periods 5 and 6. The order cost is 2736.93 monetary units and the maintenance cost is equal to 70.519 monetary units, the total cost is equal to the sum of these two and equal to 2807.449 monetary units.

- Based on the results of the implementation of Wagner-Whitney algorithm with fuzzy parameters for plastic pallet goods in two periods, the first period, 398 units (as much as demand) will be ordered and in the second period, 301 units of goods (as much as demand) will be ordered. The order cost is 968.4 monetary units and the maintenance cost is equal to 0 monetary units. In fact, according to the results of this algorithm, it is ordered for plastic pallets in each period and the maintenance cost does not include this product.

- According to the results of Wagner-Whitin algorithm with fuzzy parameters for glue in the period, the first 1000 order units, as much as the demand of both the first and second periods, the goods are ordered and the second period, no order is issued. The order cost was 417 monetary units and the maintenance cost was equal to 13.561 monetary units. The total cost of this product was equal to 430,561 units.

- According to the results of the Wagner-Whitin algorithm with fuzzy parameters for two-period color goods, in the first period, 840 units of orders will be issued as the sum of the demands of both periods, and naturally will not be customized in the second period. The cost of ordering 577.92 monetary units and the maintenance cost equal to 121.303 monetary units will be imposed on the organization. The total cost of ordering and maintaining paint products for the company is equal to 699,223 monetary units.

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